Quantum entanglement: from basic question to technological resource

Francesco Buscemi[1]

KIAS School of Computational Sciences, Online Colloqium Series

24 November 2020

[1]Nagoya University, www.quantumquia.com

Entanglement: the origin

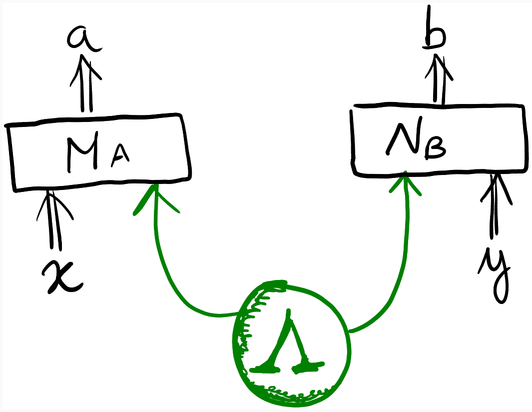
85 years of quantum entanglement

• Einstein-Podolsky-Rosen	1935	(meta)physical problem
• Schrödinger (in response to EPR)	1935-36	
⋮	⋮	⋮
• Bell	1964	physical (falsifiable) problem
⋮	⋮	⋮
• Aspect	1981	experimental fact
⋮	⋮	⋮
• Ekert	1991	cryptography!
• Bennett *et al*	1992	cryptography! superdense coding!
• Bennett *et al*	1993	quantum teleportation!
⋮		

The EPR argument

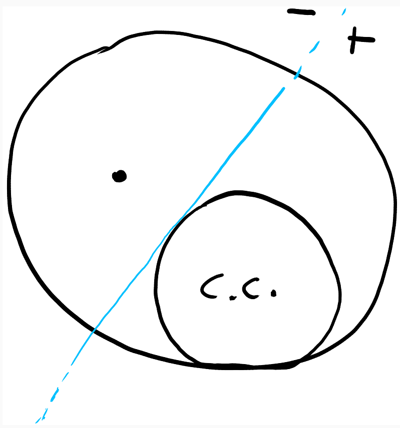
- quantum states: unit vectors in a complex vector space $\mathbb{C}^d$
- bipartite states: unit vectors in $\mathbb{C}^d \otimes \mathbb{C}^d \simeq \mathbb{C}^{d^2}$
- entangled states: $|\Phi^+\rangle_{AB} \propto |\uparrow_Z\rangle_A |\uparrow_Z\rangle_B + |\downarrow_Z\rangle_A |\downarrow_Z\rangle_B = |\uparrow_X\rangle_A |\uparrow_X\rangle_B + |\downarrow_X\rangle_A |\downarrow_X\rangle_B$
  - measuring Z_A, system B "collapses" either on $|\uparrow_Z\rangle_B$ or on $|\downarrow_Z\rangle_B$
    $\implies$ Z_B is "real"
  - measuring X_A, system B "collapses" either on $|\uparrow_X\rangle_B$ or on $|\downarrow_X\rangle_B$
    $\implies$ X_B is "real"
- A can *choose* "what is real" on B, and such "effect" happens *instantaneously*
- but because $[X, Z] \neq 0$, Z_B and X_B cannot be "real" *simultaneously* $\implies$ contradiction

Bell's formalization



- classical correlations (i.e. with a *local hidden variable model*):
  $p(a, b|x, y) = \sum_\lambda p_\Lambda(\lambda) p_A(a|x, \lambda) p_B(b|y, \lambda)$
- quantum correlations:
  $p(a, b|x, y) = \mathrm{Tr}\Big\{\Lambda_{AB}\ (M_A^{a|x} \otimes N_B^{b|y})\Big\}$

Bell inequalities



- crucial observation: the set of classical correlations is a convex subset of quantum correlations
- the blue line (a hyperplane) defines a Bell inequality

A famous example: the CHSH game

(x,y) \ (a,b)	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	◯	×	×	◯
(0,1)	◯	×	×	◯
(1,0)	◯	×	×	◯
(1,1)	×	◯	◯	×

- also known as "the XOR game": $x \cdot y = a \oplus b$
- winning probability: $\frac{1}{4}\sum p(a,b|x,y)\delta_{x\cdot y, a\oplus b} \qquad (= \vec{p}\cdot\vec{f}\,)$
- with classical correlations: max winning probability 0.75
- with quantum correlations: max winning probability $(\cos\pi/8)^2 \approx 0.85$
- with communication: win always (trivialization)

From paradox... to resource

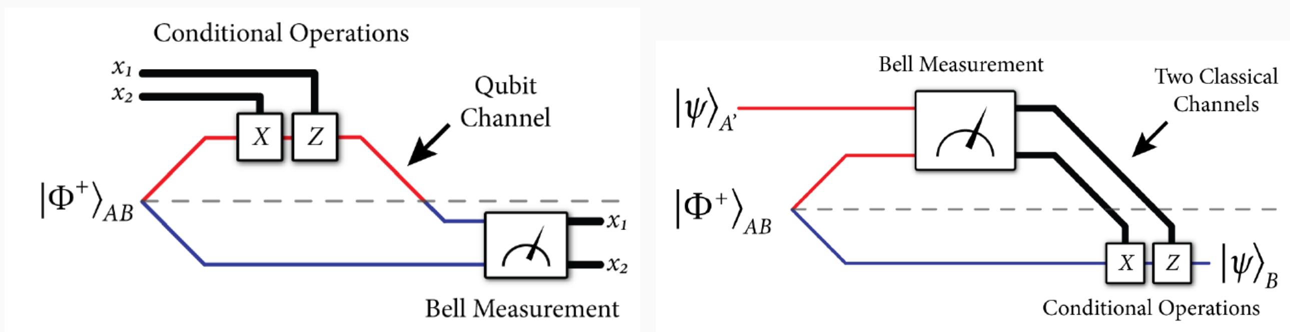


Figure 1: Superdense coding and teleportation. (Figures taken from M.M.Wilde's textbook available at https://arxiv.org/abs/1106.1445.)

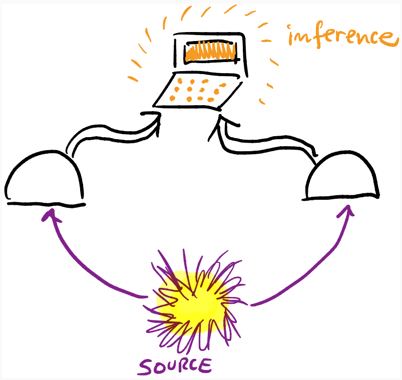
Entanglement is crucial to the design and development of quantum technologies

How can we test/benchmark entanglement?

Statistical tests of "quantumness"

How to “observe” quantum entanglement?

By *quantum tomography*



- first, reconstruct the source state from local measurements
- then, compute “how likely” it is that the source is emitting entangled signals
- technical keyword: *entanglement witnesses*

Problem with tomography

Fully trusted, *perfectly* known measuring devices are required.

What happens if we don't trust the devices?

Entanglement certification in an adversarial scenario

Any statistically relevant violation of *any* Bell inequality
$\implies$ nonclassical correlations
$\implies$ quantum entanglement

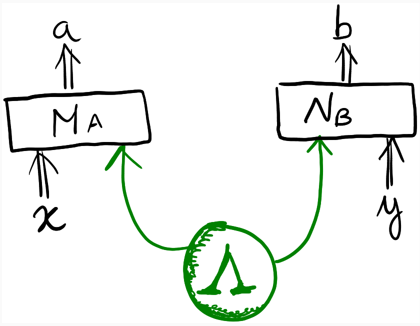
Problems with Bell tests

- communication between A and B must be ruled out
  $\rightsquigarrow$ space-like configuration
- losses in the detectors can be used to cheat
  $\rightsquigarrow$ very efficient measurements
- bar is set very high (there exist entangled states that will never pass this test)
  $\rightsquigarrow$ addition of auxiliary systems (concatenated Bell tests)
  $\rightsquigarrow$ "*it takes entanglement to certify entanglement*"

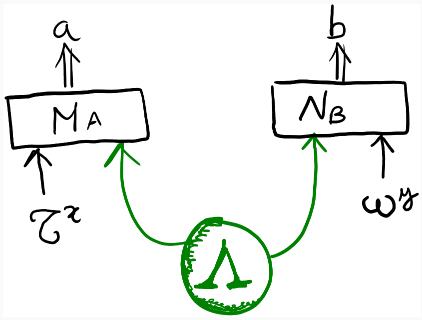
Can we trade a bit of "trust" for "practicality"?

From Bell tests to "semiquantum" tests

from this:



to this:



Pros and cons of semiquantum tests

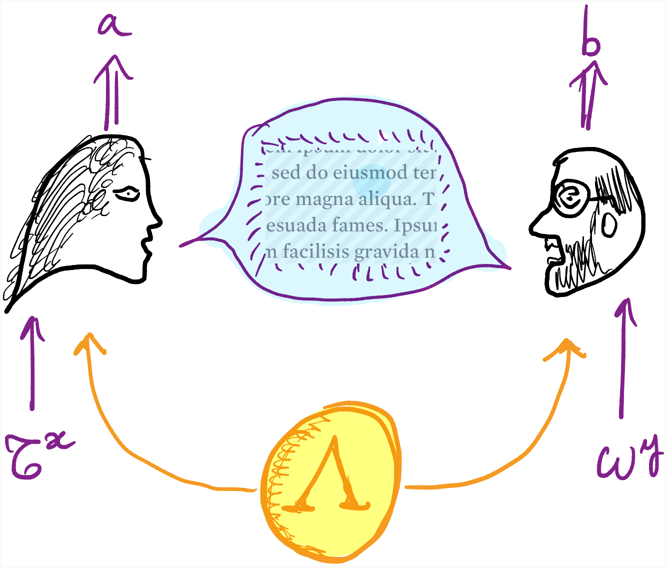
With respect to quantum tomography:

- preparation of "question states" must be trusted, but measuring devices need not (*measurement device-independent* entanglement witnesses)

With respect to Bell tests:

- semiquantum tests are able to verify *any* entangled state (i.e., they are faithful)
- semiquantum tests are robust against *classical communication* and losses

More about classical communication

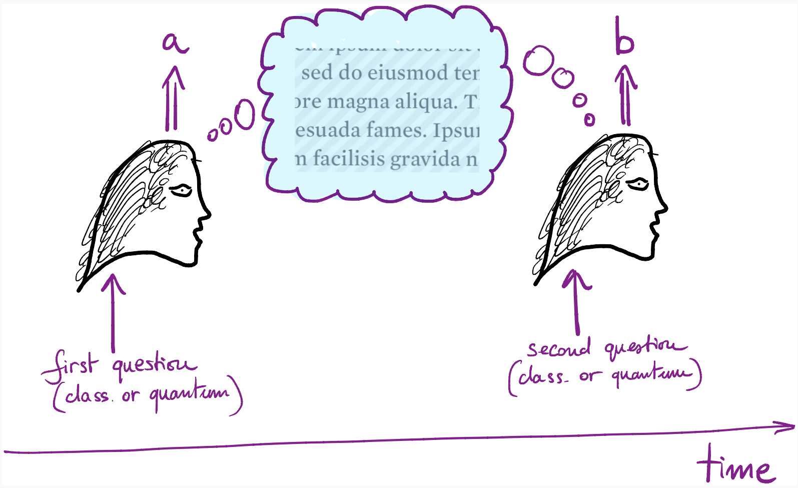


If the source is not entangled, classical communication does not help.

When is it that classical communication cannot be prevented?

When it happens in time...

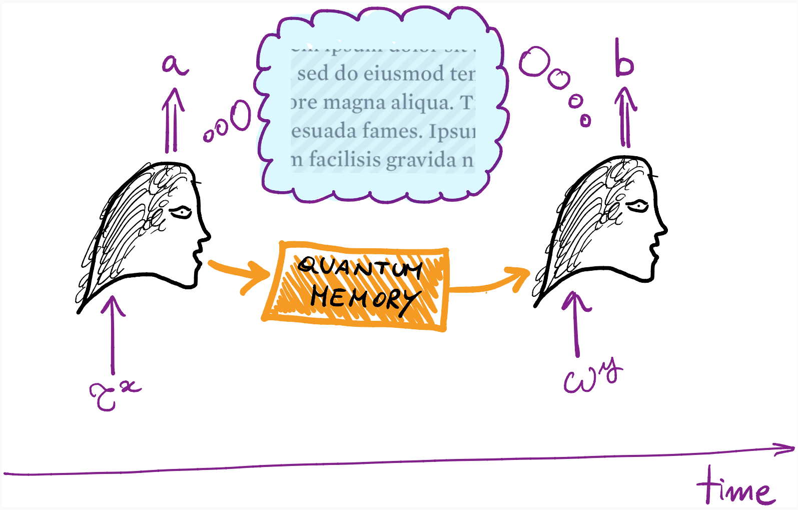
...in which case it becomes a *memory*!



in time, Bell tests become trivial

$\rightsquigarrow$ "clumsiness loophole" in Leggett-Garg tests

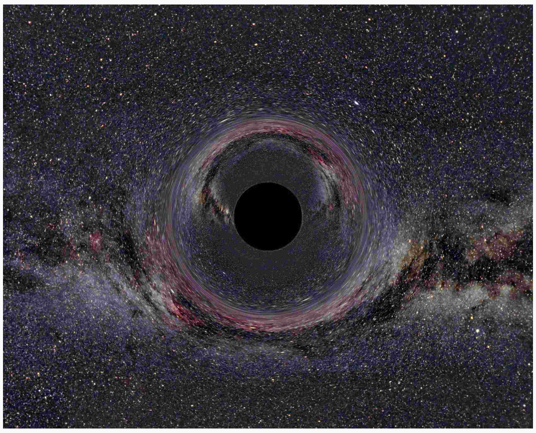
Certification of quantum memories



For any quantum memory there exists a semiquantum test (in time) that separates it from unlimited classical memory

About certification

Semiquantum tests can certify *any* form of quantum correlation between *any* two events in space-time...



...as long as the output can reach the tester!

Connections with statistical comparison theory

Statistical comparison theory

Statistical experiments:

- unknown parameter to estimate: $\theta \in \Theta$
- the value of θ influences the distribution of the sample $x \in \mathcal{X}$
- statistical dependence is described by $p(x|\theta)$

Comparison of statistical experiments:

- consider two experiments $p(x|\theta)$ and $q(y|\theta)$
- comparison: which one is more informative?

  $\rightsquigarrow$ *more informative*: "yielding better values in statistical tests"

Blackwell-Sherman-Stein Theorem (1948-1953)

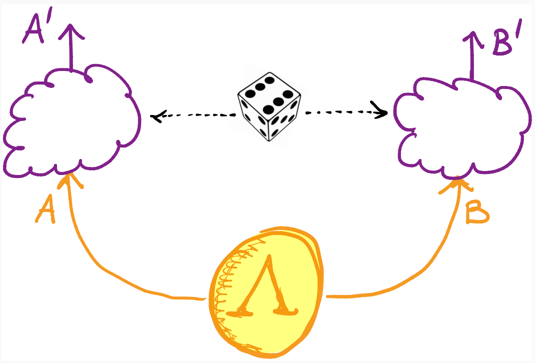
A statistical experiment $p(x|\theta)$ is more informative than $q(y|\theta)$, if and only if there exists a stochastic transformation $\lambda(y|x)$ such that

$$q(y|\theta) = \sum_x \lambda(y|x) p(x|\theta) , \qquad \forall \theta \in \Theta$$

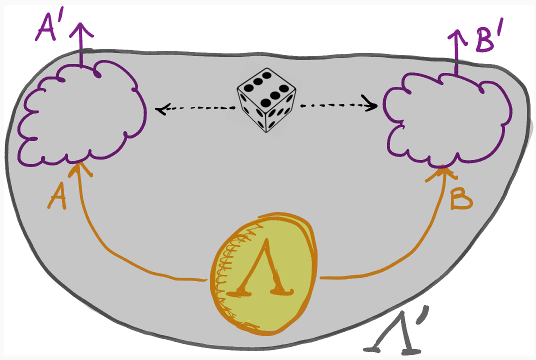
A "Blackwell Theorem" for entanglement?

"All entangled quantum states are nonlocal" (2012)

[F.B., PRL, 2012] A bipartite source Λ_{AB} yields better payoffs than $\Lambda'_{A'B'}$ in all semiquantum tests if and only if



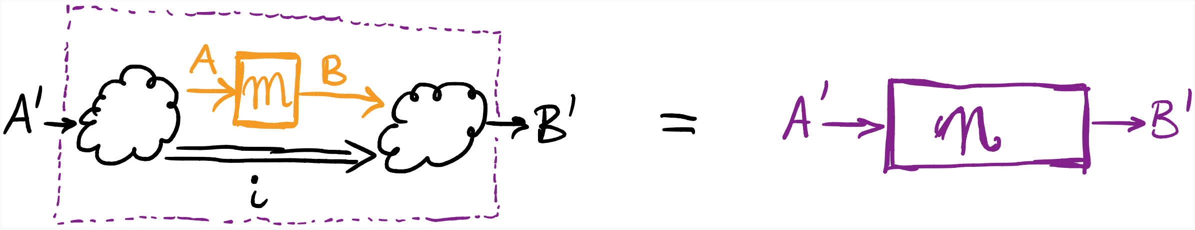
$=$



in formula: $\Lambda'_{A'B'} = \underbrace{\sum_r p(r)(\mathcal{E}^r_{A\to A'} \otimes \mathcal{F}^r_{B\to B'})}_{\text{LOSR}} \Lambda_{AB}$

The case of quantum memories

[D. Rosset, F.B., Y.-C. Liang, PRX, 2018] A quantum memory $\mathcal{M}_{A\to A'}$ yields better payoffs than $\mathcal{N}_{B\to B'}$ in all semiquantum tests if and only if

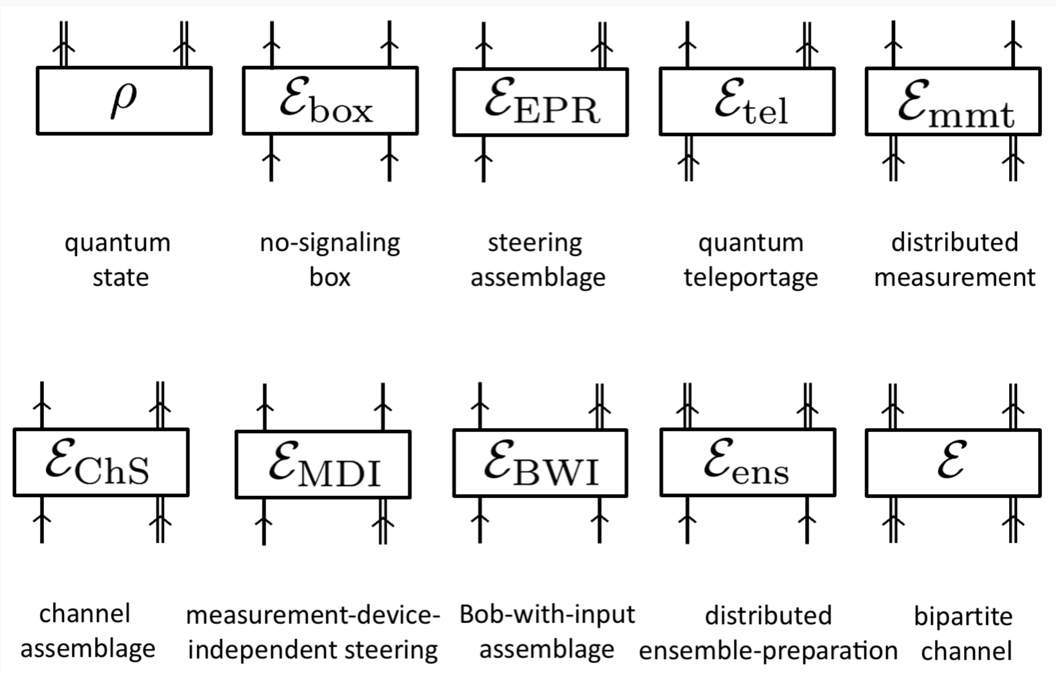


in formula: $\mathcal{N}_{B\to B'}(\cdot) = \sum_i (\mathcal{D}^i_{A'\to B'} \circ \mathcal{M}_{A\to A'} \circ \mathscr{I}^i_{B\to A})(\cdot)$

“Grand-unification” of bipartite resources (2020)

[D. Schmid, D. Rosset, F.B., Quantum, 2020; D. Rosset, D. Schmid, F.B., PRL, 2020]

image by courtesy of David Schmid



Conclusions

- quantum entanglement: the “sleeping beauty” of quantum foundations for 60 years, is now the crucial resource for quantum technologies
- tradeoffs in statistical tests for entanglement: quantum tomography vs Bell tests vs semiquantum tests
- semiquantum tests seem to hit the sweet spot
- extensions: from quantum entanglement, to quantum memories, and beyond
- maths: order-theoretic developments (statistical comparison theory)