Semiquantum Tests: Closing the "Nonlocality Gap" in Space and the "Clumsiness Loophole" in Time

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Symposium for Celebrating 60 years of Bell's theorem
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Abstract

In this presentation, I will discuss my proposal to extend nonlocal games to a semiquantum (or "measurement device-independent") framework. In this setting, questions are encoded in quantum states rather than being directly communicated to the players. I will demonstrate how this modification achieves two key objectives:

1. Bridging the gap between entanglement and nonlocality in games involving space-like separated parties.
2. Addressing the "clumsiness loophole" that affects all Legget–Garg-type tests conducted between timelike separated parties.

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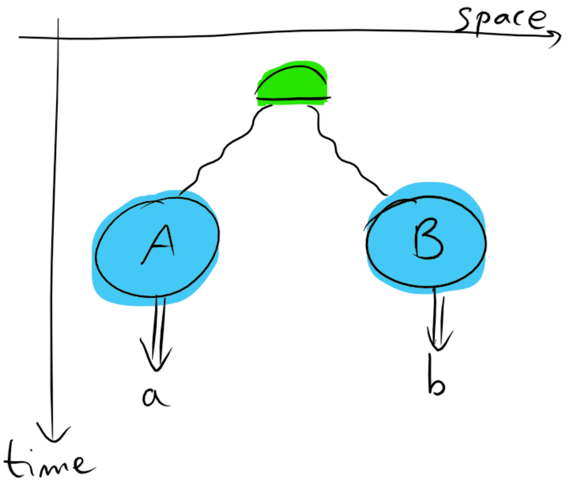
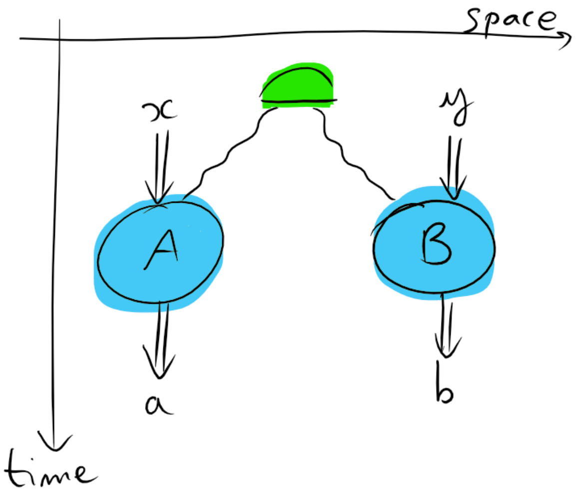
Two paradigms for entanglement verification

Entanglement witnesses



$$p(a,b) = \mathrm{Tr}\big[(P_A^a \otimes Q_B^b)\ \rho_{AB}\big]$$

Bell tests



$$p(a,b|x,y) = \mathrm{Tr}\Big[(P_A^{a|x} \otimes Q_B^{b|y})\ \rho_{AB}\Big]$$

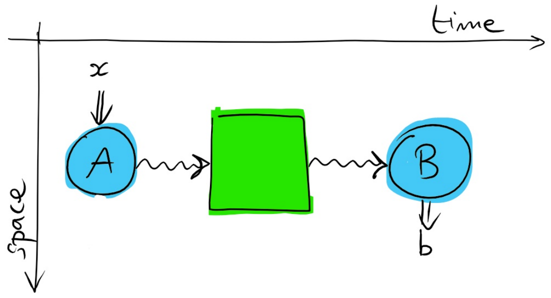
- ☺ faithfulness: for any entangled state, there exists a witness detecting it
- ☹ measurement devices need to be perfect

- ☹ hidden nonlocality: some entangled states never violate any Bell inequality
- ☺ device independence

The time-like analogue: quantum memory verification

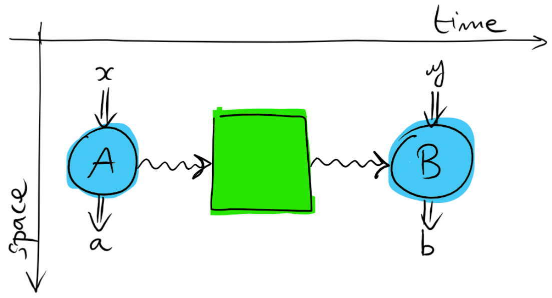
- the Choi correspondence, $\mathcal{E}_{A\to B} \longleftrightarrow \rho_{AB}$, suggests trying the same approach in time
- encouraging fact: "classical" (i.e., separable) states correspond to "classical" (i.e., entanglement-breaking) channels

Process tomography



$$p(b|x) = \mathrm{Tr}[\mathcal{E}(\sigma_x)\ P_b]$$

Time-like Bell tests

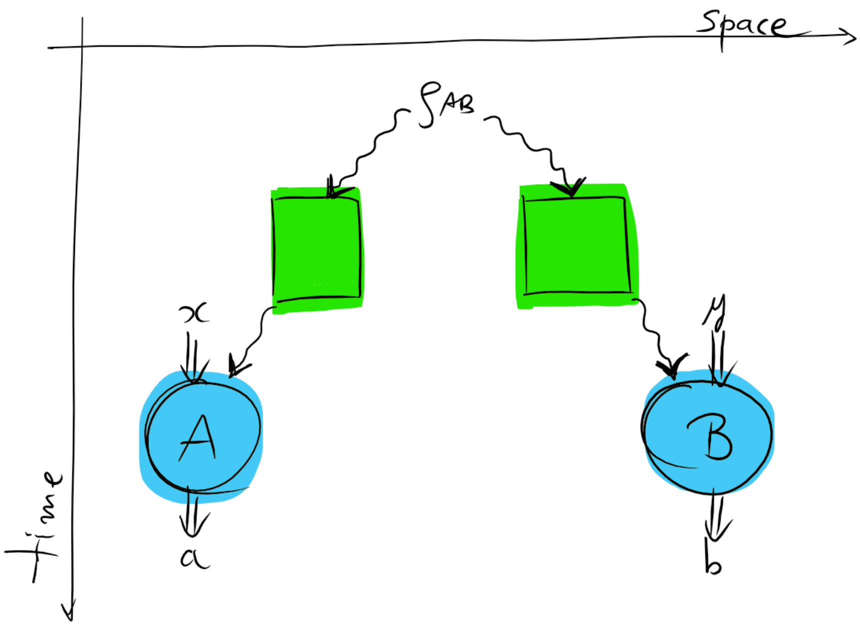


$$p(a,b|x,y) = \mathrm{Tr}\big[\mathcal{E}(\sigma_{a|x})\ P_{b|y,x,a}\big]$$

- in full analogy with entanglement witnesses, process tomography is faithful (☺) but requires complete trust in the tomographic devices (☹)
- time-like Bell tests trivialize: A can always signal to B ($\lim_{n\to\infty} ☹^{\otimes n}$)

One way around

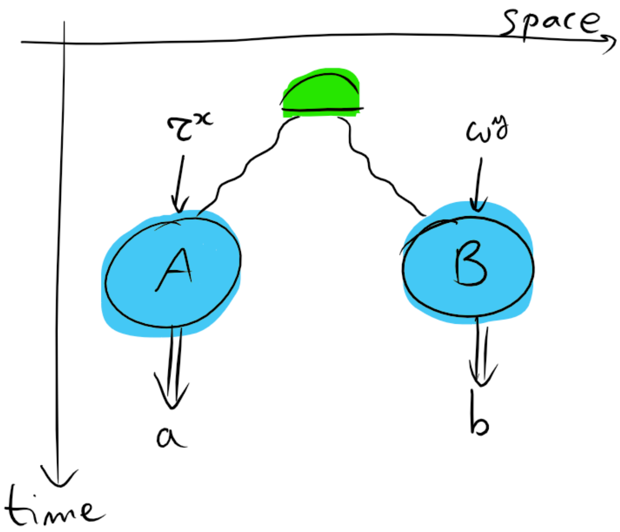
- suppose that *two* quantum memories are available: then one can imagine doing the following



- here, we need *two* quantum memories, and the test is assessing *the pair* simultaneously (and it's a Bell test, hence device-independent but not faithful)
- thus the problem remains: is it possible to certify a single given memory, without using any side-channel?

Semiquantum nonlocal games

- quantum bipartite statistical decision games, a.k.a. semiquantum games: questions are encoded on quantum states (PRL, 2012)
- the referee chooses questions x and y at random
- the referee encodes questions on quantum states $\tau^x_{A'}$ and $\omega^y_{B'}$
- the system A' is sent to Alice, B' to Bob
- Alice and Bob locally compute answers a and b

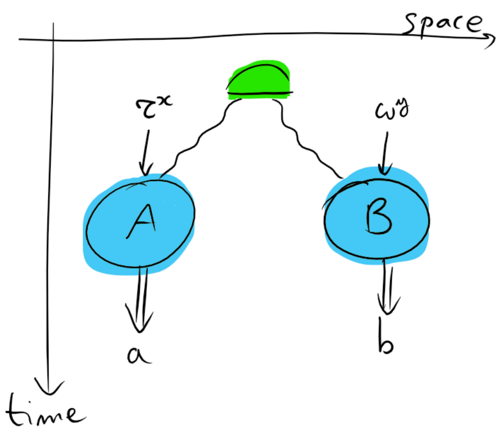


- achievable correlations are given by

$$p(a,b|x,y,\rho_{AB}) = \mathrm{Tr}\Big[(P^a_{A'A}\otimes Q^b_{BB'})\;(\tau^x_{A'}\otimes\rho_{AB}\otimes\omega^y_{B'})\Big]$$

More about semiquantum nonlocal games

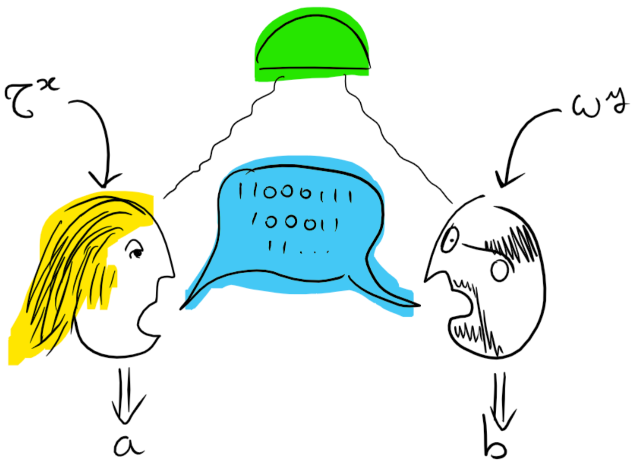
- usual Bell tests are recovered for distinguishable question states
- defining $\mathcal{P}(\rho_{AB}) = \{p(a,b|x,y,\rho_{AB})$ for some semiquantum game$\}$, we have $\mathcal{P}(\rho_{AB})\supseteq\mathcal{P}(\sigma_{CD})$ if and only if $\sigma_{CD}=\sum_i p_i(\mathcal{E}^i_A\otimes\mathcal{F}^i_B)(\rho_{AB})$
- namely, semiquantum games provide a complete set of monotones for the (pre-) ordering relation induced by "Local Operations and Shared Randomness" (LOSR)



- this implies faithfulness: for any entangled state, there is a semiquantum game detecting it
- interpretation as measurement-device-independent entanglement witnesses (Branciard et al., 2013; Cavalcanti et al., 2013): the referee needs to trust only the preparation devices in her lab
- this result is a special case of quantum statistical comparison: powerful link between statistics and dynamics (quantum thermodynamics, quantum resource theories, quantum information theory, measurements (in)compatibility, etc)

Robustness of semiquantum games against classical communication

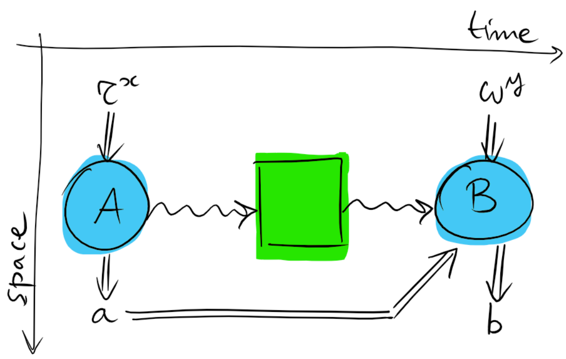
- ✔ any Bell test is spoiled, as soon as one player can communicate with the other one
- ✔ $\implies$ Bell tests cannot verify quantum channels
- ✔ Rosset et al., 2013: there exist semiquantum games that are robust against unlimited classical communication (in fact, up to any SEPP protocol)
- ✔ this feature is especially welcome in the time-like scenario, where signaling cannot be ruled out and hence *must be assumed*



$$p(a,b|x,y) = \mathrm{Tr}\Big[(P^{ab}_{\mathrm{LOCC}})\ (\tau^x_{A'} \otimes \rho_{AB} \otimes \omega^y_{B'})\Big]$$

(LOCC w.r.t. $A'A \leftrightarrow BB'$)

Time-like semiquantum games



(here we should think of B as "Alice after some time")

- ✔ give Alice a state τ^x at time t_0
- ✔ wait some time
- ✔ give her another state ω^y at time t_1
- ✔ the round ends with Alice outputting an outcome b

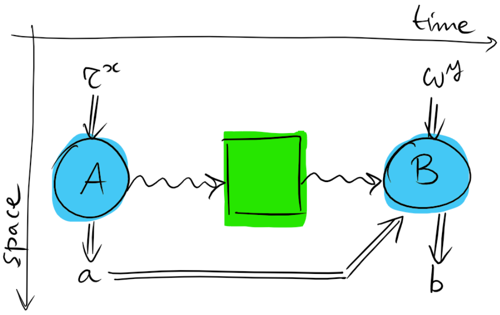
the input/output correlation is computed as

$$p(b|x,y) = \sum_a \mathrm{Tr}\Big[P^{b|a}_{BA}\ \{\omega^y_B \otimes \mathcal{E} \circ \mathcal{I}^a(\tau^x_A)\}\Big]$$

where $\{\mathcal{I}^a\}$ is an instrument, so that any amount of classical communication can be transmitted through the index a

Features of time-like semiquantum games

- ✔ as long as the quantum memory (channel) $\mathcal{E}$ is not entanglement breaking, there exists a time-like semiquantum game capable of certifying that
- ✔ assumption: we need to trust the preparation of states τ^x and ω^y, but that is anyway required in the time-like scenario (no fully device-independent quantum channel verification [Pusey, 2015])
- ✔ $\Longrightarrow$ faithfulness with minimal assumptions
- ✔ extra feature: it is possible to *quantify* the minimal dimension of the quantum memory



Conclusions

- ✔ entanglement witnesses: faithful, but complete trust is necessary
- ✔ Bell tests: fully device-independent, but not faithful
- ✔ semiquantum tests: faithful, and trust is required only for the referee’s preparation devices
- ✔ semiquantum tests are particularly compelling in the time-like scenario, in which no device-independent quantum channel verification exists anyway
- ✔ $\Longrightarrow$ verification of non-classical correlations among any two locally quantum agents, independent of their causal separation
- ✔ the test is quantitative: a lower bound on the quantum dimension can be given

fine

References

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2. D. Rosset, F. Buscemi, and Y.-C. Liang, *Resource Theory of Quantum Memories and Their Faithful Verification with Minimal Assumptions.* Physical Review X, vol. 8, 021033 (2018).