Open quantum systems, non-unitarity, and non-Markovian dynamics

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Observing a Century of Quantum Mechanics
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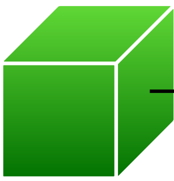
Closed systems evolution

According to Schrödinger's equation, closed (isolated) quantum systems evolve by unitary evolution:

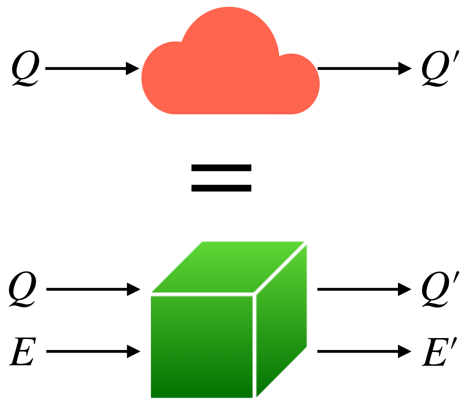
$$|\psi'(t_1)\rangle = U(t_0 \to t_1)|\psi(t_0)\rangle \ , \qquad U^\dagger U = \mathbb{1} \ .$$

$|\psi\rangle \longrightarrow$  $\longrightarrow |\psi'\rangle$

Markov property: the state of the system at time t_1 depends only on the state of the system at time t_0.

Open systems evolution

Closability assumption: any "open system" Q can be "closed" by taking into account all the parts of the universe (i.e., the "environment" E) that interact with it



But what about the initial joint state?
And the Markov property?

The problem of initial correlations in a nutshell

Textbooks usually begin with the factorization assumption, i.e., $\bullet_Q \otimes \gamma_E$.

In this case, the reduced dynamics (i.e., the "open system's dynamics") is always well defined, completely positive and trace-preserving – it is a quantum channel

$$\mathrm{Tr}_{E'}\left[U_{QE\to Q'E'}\,(\bullet_Q \otimes \gamma_E)\,U^\dagger_{QE\to Q'E'}\right] =: \mathcal{E}_{Q\to Q'}(\bullet_Q)\ .$$

However:

- 1994: Pechukas' PRL (what if we drop the factorization assumption?) and Alicki's comment on it
- 2004: Sudarshan's group (explicit constructions and examples)
- 2009: Shabani and Lidar's PRL (claim: quantum discord solves the problem)
- 2013: Brodutch *et al.*'s counterexample voiding the Shabani–Lidar PRL

The assignment map approach (Pechukas–Alicki)

The initial conditions should be given as a linear, completely positive map $\mathcal{A} : Q \to QE$ satisfying the consistency relation $\mathrm{Tr}_E[\mathcal{A}(\bullet_Q)] = \bullet_Q$

$$
\begin{tikzcd}
QE \arrow[r, "U"] \arrow[d, shift left, "\mathrm{Tr}_E"] & Q'E' \arrow[d, "\mathrm{Tr}_{E'}"'] \\
Q \arrow[u, shift left, "\mathscr{A}"] \arrow[r, dotted] & Q'
\end{tikzcd}
$$

However, the consistency requirement is very restrictive: "natural" interactions that create correlations between Q and E almost never satisfy it.

The preparability approach

FB, PRL 2014

Let us denote the set of possible initial system-environment states σ_{QE} by $\mathfrak{S}_{QE} \subseteq \mathcal{S}(\mathcal{H}_Q \otimes \mathcal{H}_E)$.

The set $\mathfrak{S}_{QE}$ is said to be *preparable* if and only if there exists an input system R and a CP linear map $\mathcal{P} : R \to QE$ such that $\mathfrak{S}_{QE}$ is the filter of $\mathcal{S}(\mathcal{H}_R)$ under $\mathcal{P}$, that is,

$$\mathfrak{S}_{QE} = \mathcal{P}(\mathcal{S}(\mathcal{H}_R)) := \left\{ \frac{\mathcal{P}(\varrho_R)}{\mathrm{Tr}[\mathcal{P}(\varrho_R)]} : \varrho_R \in \mathcal{S}(\mathcal{H}_R) \ \wedge \ \mathrm{Tr}[\mathcal{P}(\varrho_R)] > 0 \right\} .$$

The set $\mathfrak{S}_{QE}$ is preparable if and only if it is *steerable*, i.e., if and only if there exists a reference system R and a tripartite density operator ω_{RQE} such that

$$\forall \sigma_{QE} \in \mathfrak{S}_{QE} \ , \ \exists \pi_R \geqslant 0 \ : \ \sigma_{QE} = \frac{\mathrm{Tr}_R[\omega_{RQE} \ (\pi_R \otimes \mathbb{1}_{QE})]]}{\mathrm{Tr}[\omega_{RQE} \ (\pi_R \otimes \mathbb{1}_{QE})]} \ .$$

CPTP reducibility

The set $\mathfrak{S}_{QE}$ is said to be *CPTP-reducible* if and only if for any interaction $U : QE \to Q'E'$, there exists a quantum channel $\mathcal{E} : Q \to Q'$ such that

$$\mathrm{Tr}_{E'}\left[U\sigma_{QE}U^\dagger\right] = \mathcal{E} \circ \mathrm{Tr}_E[\sigma_{QE}] \ , \quad \forall \sigma_{QE} \in \mathfrak{S}_{QE} \ .$$

Result

Let the set $\mathfrak{S}_{QE}$ of initial system-environment conditions be preparable/steerable. The following are equivalent:

- $\mathfrak{S}_{QE}$ is CPTP-reducible
- $\mathfrak{S}_{QE}$ is Markov-steerable: there exists a tripartite state ω_{RQE} with $I(R;E|Q)_\omega = 0$, such that $\mathfrak{S}_{QE}$ is steerable from ω_{RQE}

The reduced open system's dynamics can remain well-defined even in the presence of initial correlations between the system and the surrounding environment.

Examples

No correlations:

- fix one environment state σ_E
- define $\mathfrak{S}_{QE} = \{\varrho_Q \otimes \sigma_E : \varrho_Q \in \mathcal{S}(\mathcal{H}_Q)\}$
- in this case, $\omega_{RQE} = \Psi^+_{RQ} \otimes \sigma_E$
- $\implies I(R;E|Q)_\omega = 0 \implies$ CPTP-reducible

Classical correlations (Rodriguez-Rosario *et al.*, 2008):

- fix N environment states: $\sigma_E^{(1)}, \sigma_E^{(2)}, \ldots, \sigma_E^{(N)}$
- define $\mathfrak{S}_{QE} = \left\{\varrho_{QE} = \sum_{i=1}^N p_i |i\rangle\langle i|_Q \otimes \sigma_E^{(i)} : \{p_i\}_i \text{ prob. dist.}\right\}$
- $\omega_{RQE} = N^{-1} \sum_{i=1}^N |i\rangle\langle i|_R \otimes |i\rangle\langle i|_Q \otimes \sigma_E^{(i)}$
- $\implies I(R;E|Q)_\omega = 0 \implies$ CPTP-reducible

Can there be more?

No! Shabani and Lidar (2009) published a paper claiming that the condition of null discord would be, not only sufficient, but also necessary for CPTP reducibility.

Yes! The above claim was disproved by the following counterexample (Brodutch *et al.*, 2013).

In our formalism:

- fix three distinct environment states $\sigma_E^{(0)}$, $\sigma_E^{(1)}$, and $\sigma_E^{(2)}$
- fix two system-environment states, α and β as follows:

$$\alpha_{QE} = \frac{1}{2}|0\rangle\langle 0|_Q \otimes \sigma_E^{(0)} + \frac{1}{2}|+\rangle\langle +|_Q \otimes \sigma_E^{(1)}\ , \qquad \beta_{QE} = |2\rangle\langle 2|_Q \otimes \sigma_E^{(2)}$$

- $\mathfrak{S}_{QE} = \left\{\varrho^p_{QE} = p\alpha_{QE} + (1-p)\beta_{QE} : \forall p \in [0,1]\right\}$
- $\omega_{RQE} = \frac{1}{2}|0\rangle\langle 0|_R \otimes \alpha_{QE} + \frac{1}{2}|1\rangle\langle 1|_R \otimes \beta_{QE}$
- $\implies I(R;E|Q)_\omega = 0 \implies$ CPTP-reducible

More general examples

All counterexamples to the factorization condition only involve separable correlations.

Can we have CPTP-reducible entanglement?

Yes! Starting from tripartite states with $I(R;E|Q)_\omega = 0$, it is easy to construct a lot of counterexamples.

☞ However, there is a tradeoff between the "strength" of the correlations and the "size" of the possible initial state space of the system. For example, if we require that $\mathfrak{S}_Q := \mathrm{Tr}_E[\mathfrak{S}_{QE}] = \mathcal{S}(\mathcal{H}_Q)$, then the factorization condition is the only one that works.

What happens when the set of possible initial correlated states is not CPTP-reducible/Markov-steerable?

Information revivals

The Markov condition $I(R;E|Q)_\omega = 0$ is equivalent to the condition that, for all joint evolutions $U : QE \to Q'E'$,

$$I(R;Q)_\omega \geqslant I(R;Q')_{\omega'} \ ,$$

where $\omega'_{RQ'E'} := (\mathbb{1}_R \otimes U_{QE})\omega_{RQE}(\mathbb{1}_R \otimes U_{QE})^\dagger$. In other words, the data-processing inequality is never violated, also in the presence of initial correlations.

This is because

$$I(R;Q')_{\omega'} \leqslant I(R;Q'E')_{\omega'} = I(R;QE)_\omega = I(R;Q)_\omega + I(R;E|Q)_\omega = I(R;Q)_\omega \ .$$

Hence, if $I(R;E|Q)_\omega > 0$, i.e., if the initial set of correlations is not CPTP-reducible, information revivals can occur, i.e.

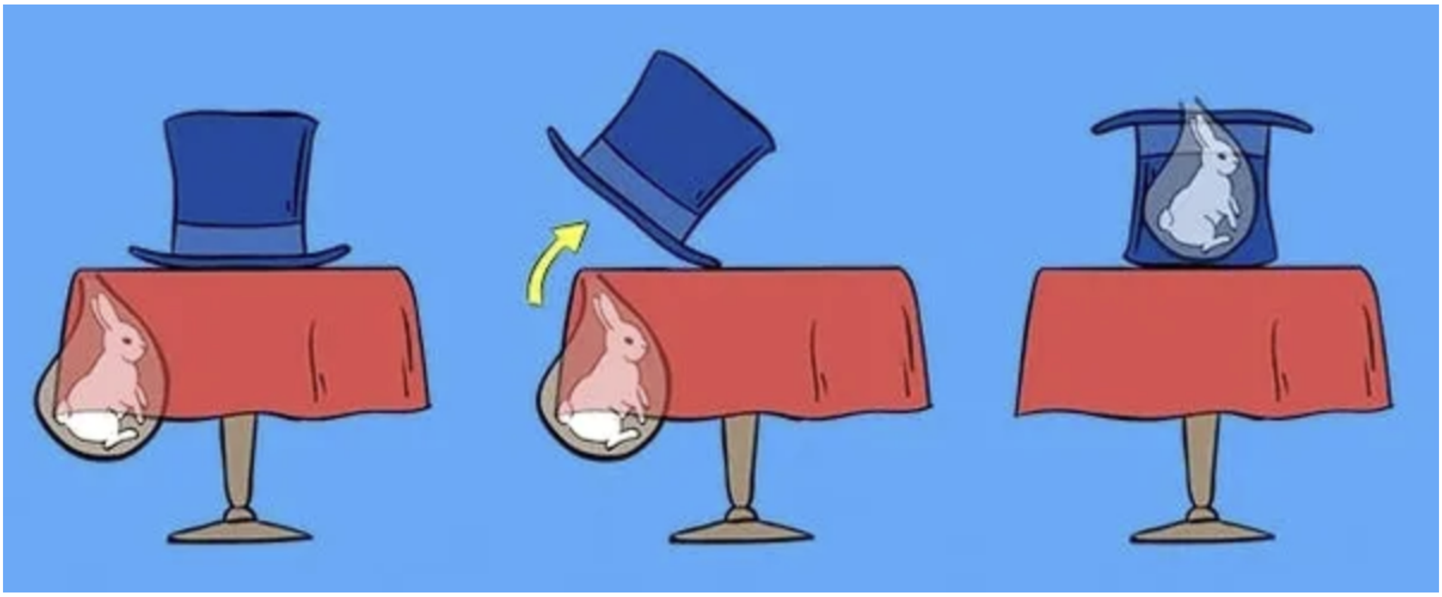
$$I(R;Q)_\omega < I(R;Q')_{\omega'} \ .$$

An information revival is a violation of locality!

It urges an explanation.

Explaining revivals

Explaining ~~revivals~~ magic tricks



Casual explanations

FB, R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, M. N. Bera; arXiv:2405.05326

Suppose that we have a revival: $I(R;Q)_\omega < I(R;Q')_{\omega'}$.

Explanation. Other parts ("regions") of the universe are added to Q', until the revival disappears, i.e., $I(R;Q\cdots)_\omega \geqslant I(R;Q'\cdots)_{\omega'}$.

Causal consistency. We need to find an extension ω_{RQE} of ω_{RQ} and a unitary operator $U: QE \to Q'E'$ such that $\omega'_{RQ'E'} = (\mathbb{1}_R \otimes U_{QE})\omega_{RQE}(\mathbb{1}_R \otimes U_{QE})^\dagger$ is an extension of $\omega'_{RQ'}$.

A causal explanation always exists

Since $\omega_R = \omega'_R$, there exists a purification $|\Psi\rangle_{RQE}$ and a unitary $U: QE \to Q'E'$ such that $\omega_{RQ'} = \mathrm{Tr}_{E'}\left[\omega'_{RQ'E'}\right]$ and $I(R;QE)_\omega = I(R;Q'E')_{\omega'}$. In general, causal explanations are not unique.

A causal explanation is also known as information backflow.

Any information revival can be explained as an information backflow.

But is a backflow always necessary?

A motivating example

- With $\mathcal{H}_R \cong \mathcal{H}_Q \cong \mathbb{C}^2$ and $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, consider the revival situation

$$\omega_{RQ} = \frac{\mathbb{1}_R}{2} \otimes \frac{\mathbb{1}_Q}{2} \,, \qquad \omega'_{RQ'} = |\Phi^+\rangle\langle\Phi^+|_{RQ'} \,.$$

- Causal explanation ($S^{wap}_{Q\leftrightarrow E}$ denotes the swap operator):

$$\omega_{RQE} = |\Phi^+\rangle\langle\Phi^+|_{RE} \otimes \frac{\mathbb{1}_Q}{2} \,, \qquad \omega'_{RQ'E'} = (\mathbb{1}_R \otimes S^{wap}_{Q\leftrightarrow E})\omega_{RQE}(\mathbb{1}_R \otimes S^{wap}_{Q\leftrightarrow E}) \,.$$

  This is a backflow and $I(R;QE)_\omega = I(R;Q'E')_{\omega'}$.

- Alternative explanation:

$$\omega_{RQEF} = \sum_{i=0}^{3} (\mathbb{1} \otimes \sigma^i_Q)|\Phi^+\rangle\langle\Phi^+|_{RQ}(\mathbb{1}_R \otimes \sigma^i_Q) \otimes |i\rangle\langle i|_E \otimes |i\rangle\langle i|_F \,,$$

$$\omega'_{RQ'E'F} = (\mathbb{1}_R \otimes C_{QE} \otimes \mathbb{1}_F)\omega_{RQEF}(\mathbb{1}_R \otimes C_{QE} \otimes \mathbb{1}_F)^\dagger \,,$$

  where $C_{QE} = \sum_{j=0}^{3} \sigma^j_Q \otimes |j\rangle\langle j|_E$.

This is an explanation because $I(R;QF)_\omega = I(R;Q'F)_{\omega'}$, but this is not a backflow!

Non-causal explanations

Consider a revival

$$\omega_{RQ} \to \omega'_{RQ'} \qquad I(R;Q)_\omega < I(R;Q')_{\omega'} \ .$$

If there exists an extension ω_{RQEF} and a unitary $U : QE \to Q'E'$ such that

$$\mathrm{Tr}_{E'F}\left[(\mathbb{1}_R \otimes U_{QE} \otimes \mathbb{1}_F)\omega_{RQEF}(\mathbb{1}_R \otimes U_{QE} \otimes \mathbb{1}_F)^\dagger\right] = \omega'_{RQ'} \ ,$$

and

$$I(R;QF)_\omega \geqslant I(R;Q'F)_{\omega'} \ ,$$

we say that the revival is non-causal.

This is because the extension F never interacts with the system: it may reside in a causally separated region of the universe, outside the causal past of Q'.

When is a causal backflow absolutely necessary?

Sufficient condition for a causal backflow

- Suppose that there is a revival, i.e., $I(R;Q)_\omega < I(R;Q')_{\omega'}$.
- A causal explanation is the only possible explanation for the revival if and only if

$$\forall F: \text{ non-causal extensions }, \quad I(R;QF)_\omega < I(R;Q'F)_{\omega'} .$$

- A sufficient condition for the above is

$$\sup_F I(R;Q|F)_\omega < \inf_F I(R;Q'|F)_{\omega'} .$$

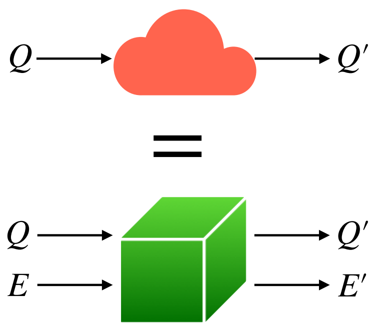
- In turn, the above holds if

$$H(Q)_\omega < E_{sq}(\omega'_{RQ'}) ,$$

  where E_{sq} denotes the squashed entanglement.
- For example: the revival $\omega_{RQ} = \frac{1}{2}\mathbb{1}_R \otimes |0\rangle\langle 0|_Q \to \omega'_{RQ'} = |\Phi^+\rangle\langle\Phi^+|_{RQ'}$ can only be explained by means of a backflow.

Non-causal correlations

Let us go back to the problem of initial system–environment correlations, where the set $\mathfrak{S}_{QE}$ of possible initial conditions is steerable from ω_{RQE}.



If we can find an extension ω_{RQEF} such that $I(R;E|QF)_\omega = 0$, then, for any unitary interaction $U : QE \to Q'E'$,

$$I(R;QF)_\omega \geqslant I(R;Q'F)_{\omega'} .$$

In other words, revivals may occur, but they will all be non-causal (i.e., no backflow).

Classification of initial correlations

A hierarchy of possibilities

Let the set $\mathfrak{S}_{QE}$ of initial system-environment conditions be steerable from ω_{RQE}.

- $I(R;E)_\omega = 0 \rightsquigarrow$ no correlations $\rightsquigarrow$ the textbook case.
- $I(R;E|Q)_\omega = 0 \rightsquigarrow$ Markov correlations $\rightsquigarrow$ system–environment correlations are present but don't cause any revival.
- $\exists$ extension ω_{RQEF} s.t. $I(R;E|QF)_\omega = 0 \rightsquigarrow$ non-causal correlations $\rightsquigarrow$ revivals can happen, but they can all be explained without a causal backflow.

Added bonus: closure under convex mixtures

- Consider two situations steerable initial conditions, described by ω_{RQE} and τ_{RQE}.
- Suppose that they are both Markov: $I(R;E|Q)_\omega = I(R;E|Q)_\tau = 0$.
- Their convex combination in general is not Markov: $I(R;E|Q)_{p\omega+(1-p)\tau} > 0$.
- However, suppose now that they are both non-causal, i.e., there exist extensions ω_{RQEF} and τ_{RQEF} such that $I(R;E|QF)_\omega = I(R;E|QF)_\tau = 0$.
- Their convex combination is automatically non-causal!

Without distinguishing between causal and non-causal information revivals, spurious "classical" randomness may be erroneously counted as a backflow.

Conclusion

Take-home ideas

- In open quantum systems dynamics, the separation is not only "revival occurs" (non-Markov) VS "revival does not occur" (Markov).
- Within revivals, we can further distinguish between "non-causal revivals" VS "genuine backflows".
- If $H(Q') < E_{sq}(R;Q'')$, then genuine backflow.
- Such "genuine non-Markovianity" is well-behaved under convex mixtures of processes $\Longrightarrow$ resource theory of genuine non-Markovianity.
- Situation analogous to the separation of total correlations in entanglement (monogamous) and classical correlations (broadcastable).

Thank you

References

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2. F. Buscemi, R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, and M.N. Bera, *Information revival without backflow: non-causal explanations of non-Markovianity*. Preprint arXiv:2405.05326, 2024.