A New Regime of Non-Markovianity: Non-Causal Revivals VS Genuine Backflows

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Abstract

It is well known that convex combinations of Markov processes typically result in non-Markov ones. In this talk I will review some notions of (non-)Markovianity for quantum stochastic processes focusing in particular on a recent proposal to quantify information backflows after classical memories have been suitably "squashed" out. Such a squashed non-Markovianity, besides suggesting a notion of genuine (or causal) information revivals, is also able to resolve the problem of non-convexity, thus clarifying the role of genuine non-Markovianity as a resource for quantum information processing.

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Non-Causal Revivals VS Genuine Backflows

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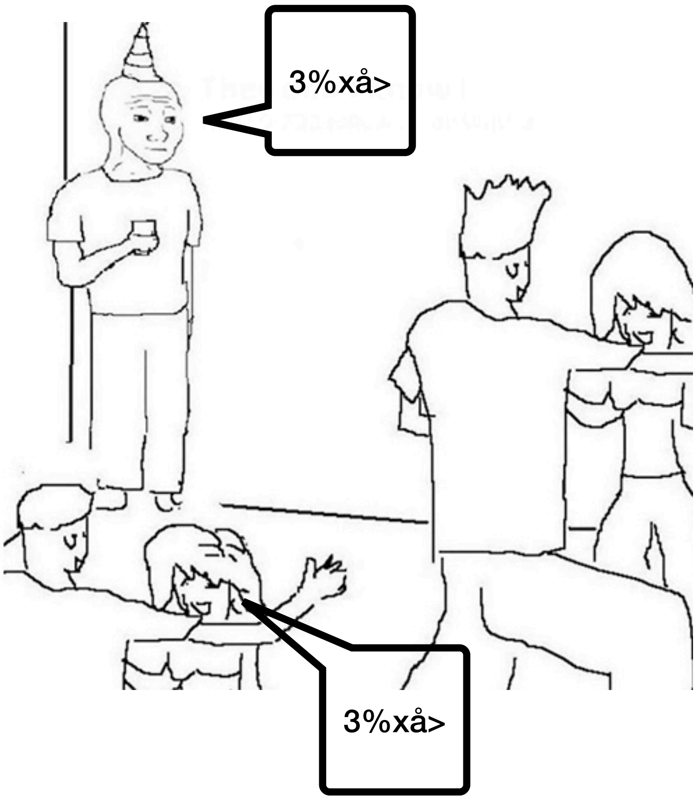
References

- F.B.: *On complete positivity, Markovianity, and the quantum data-processing inequality, in the presence of initial system-environment correlations.*
  Physical Review Letters 113, 140502 (2014)

- F.B., R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, M.N. Bera: *Information revival without backflow: non-causal explanations of non-Markovianity.*
  PRX Quantum 6, 020316 (2025)

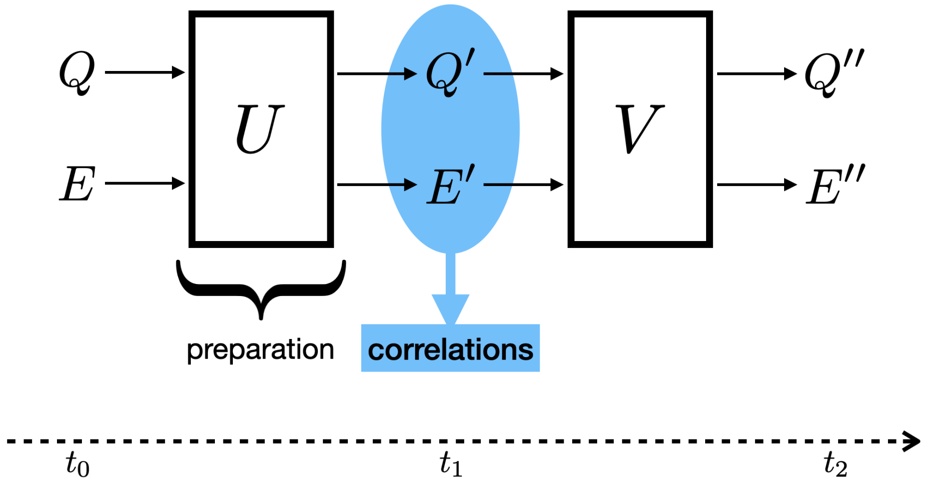
There are echoes and echoes...



The first may just be an "accidental" echo, the second is most probably a "causal" echo...

Does this simple idea carry over to information revivals in open quantum systems dynamics?

Information revival 1/2

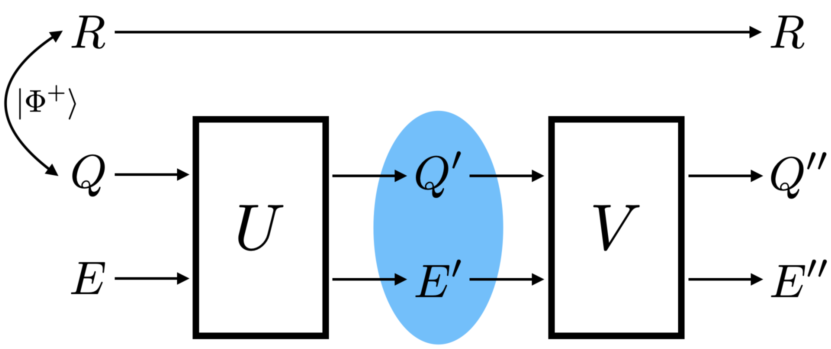


$$\mathcal{S}(\mathcal{H}_Q)\otimes\gamma_E \xrightarrow{U_{QE}:t_0\to t_1} \mathfrak{S}_{Q'E'} \xrightarrow{V_{Q'E'}:t_1\to t_2} \mathfrak{S}_{Q''E''}$$

Information revival 2/2

for convenience, we introduce a reference system $\mathcal{H}_R \cong \mathcal{H}_Q$ and a maxent state $|\Phi^+\rangle_{RQ}$

$$\Phi^+_{RQ}\otimes\gamma_E \xrightarrow{t_0\to t_1} \underbrace{U_{QE}(\Phi^+_{RQ}\otimes\gamma_E)U^\dagger_{QE}}_{\equiv\sigma_{RQ'E'}} \xrightarrow{t_1\to t_2} \underbrace{V_{Q'E'}\sigma_{RQ'E'}V^\dagger_{Q'E'}}_{\equiv\tau_{RQ''E''}}$$

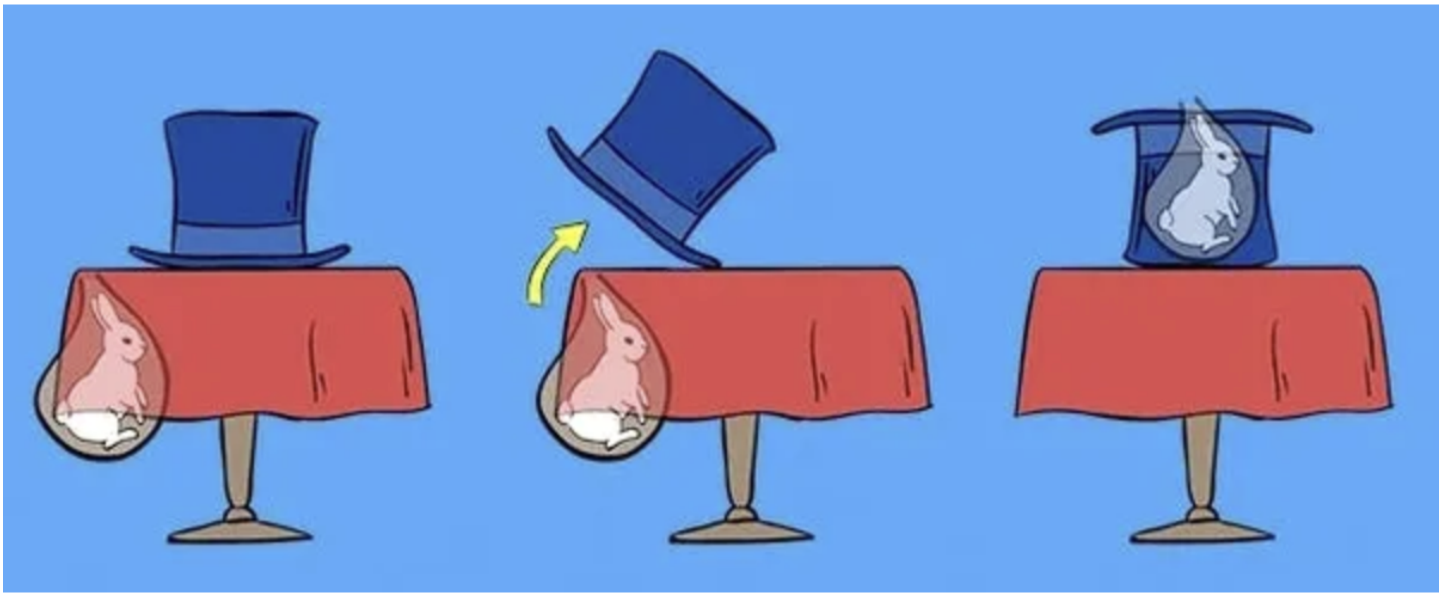


if $I(R;E'|Q')>0$, a revival, may occur: i.e., we could have $I(R;Q'')>I(R;Q')$

By looking at the system alone, a revival amounts to a violation of locality!

As such, it needs an explanation.

Explaining revivals



Explaining revivals

Suppose that a revival happens between t_1 and t_2, i.e., $I(R;Q'') > I(R;Q')$, and consider an interaction model such as:

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow[t_0 \to t_1]{U_{QE}} \sigma_{RQ'E'} \xrightarrow[t_1 \to t_2]{V_{Q'E'}} \tau_{RQ''E''} \qquad (1)$$

Explanation: compatibly with the model (1), keep adding parts of the universe to Q', until the revival disappears, i.e., $I(R;Q'\cdots) \geqslant I(R;Q''\cdots)$

Obvious explanation: just add the environment itself! Indeed, due to unitarity, $I(R;Q'E') = I(R;Q''E'')$

$\Longrightarrow$ **causal information backflow**

Statement no. 1

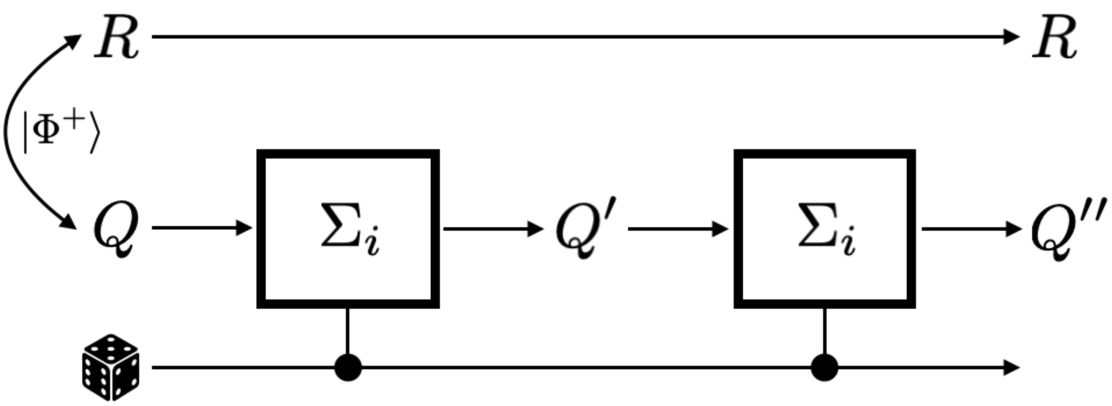
If $I(R;E'|Q') > 0$, information revivals are possible.

Any revival can be explained as a backflow.

But is a backflow always necessary?

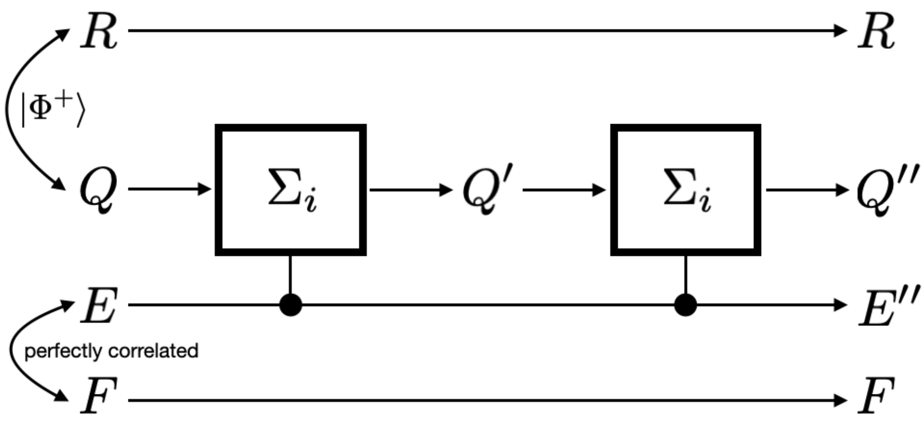
A motivating example 1/2

$\mathcal{H}_R \cong \mathcal{H}_Q \cong \mathbb{C}^2$, $\gamma_E = \frac{1}{4}\mathbb{1}$, $\Sigma^i \in \{\mathbb{1}, X, Y, Z\}$



$$I(R;Q) = 2 \xrightarrow{t_0 \to t_1} I(R;Q') = 0 \xrightarrow{t_1 \to t_2} I(R;Q'') = 2$$

A motivating example 2/2



$$I(R;QF) = 2 \xrightarrow{t_0 \to t_1} I(R;Q'F) = 2 \xrightarrow{t_1 \to t_2} I(R;Q''F) = 2$$

$\Longrightarrow$ F provides an explanation, even though it never interacts with Q and is causally separated from it at all times!

$\Longrightarrow$ there **cannot** be any "backflow" from F into Q

Statement 2

A backflow is not always necessary to explain information revivals.

So, when is a backflow absolutely necessary?

And when instead do we never need one?

Sufficient condition for a causal backflow

- suppose that there is a revival, i.e., $I(R;Q') < I(R;Q'')$
- a non-causal explanation does not exists if and only if

$$\forall F: \text{ non-causal extensions }, \quad I(R;Q'F) < I(R;Q''F)$$

- a sufficient condition for the above is

$$\sup_F I(R;Q'|F) < \inf_F I(R;Q''|F)$$

- in turn, the above holds if

$$H(Q')_\sigma < E_{sq}(\tau_{RQ''})$$

Statement 3

If $H(Q')_\sigma < E_{sq}(\tau_{RQ''})$, then the revival requires a backflow, regardless of the interaction model.

Are there processes that never require a backflow?

Non-causal correlations

$$\Phi^+_{RQ} \otimes \gamma_{EF} \xrightarrow{U_{QE}} \sigma_{RQ'E'F} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''F}$$

$$I(R;Q'F) \geqslant I(R;Q''F) \iff I(R;E''|Q''F) \geqslant I(R;E'|Q'F)$$

non-causal correlations: if there exists a causally separated extension F such that $I(R;E'|Q'F) = 0$, the system-environment correlations present at time $t = t_1$ are called non-causal

Statement 4

If there exists F such that $I(R;E'|Q'F) = 0$, revivals may occur, but they can only be non-causal: a backflow is never required.

Added bonus: closure under convex mixtures

take the mixture of two processes

$$p\left\{\Phi^+_{RQ}\otimes\gamma^{(a)}_E \to \sigma^{(a)}_{RQ'E'} \to \tau^{(a)}_{RQ''E''}\right\} + (1-p)\left\{\Phi^+_{RQ}\otimes\gamma^{(b)}_E \to \sigma^{(b)}_{RQ'E'} \to \tau^{(b)}_{RQ''E''}\right\}$$

even if both $\sigma^{(a)}_{RQ'E'}$ and $\sigma^{(b)}_{RQ'E'}$ only contain Markov correlations, their mixture could allow revivals, i.e.,

$$I(R;E'|Q')_a = 0 \ \wedge \ I(R;E'|Q')_b = 0 \quad \not\Longrightarrow \ I(R;E'|Q')_{pa+(1-p)b} = 0$$

instead, any convex mixture of non-causal correlations is automatically non-causal

$\Longrightarrow$ we can construct a convex resource theory of **genuine** (causal) **non-Markovian backflows**

Conclusion

Today's take-home ideas

- in open quantum systems dynamics, the separation is not only "revival does not occur" (Markov) VS "revival occurs" (non-Markov)
- within revivals, we can further distinguish between "non-causal revivals" VS "genuine backflows"
- $\exists F$ such that $I(R; E'|Q'F) = 0 \implies$ non-causal revivals only
- $H(Q') < E_{sq}(R; Q'') \implies$ genuine backflow
- such "genuine non-Markovianity" is well-behaved under convex mixtures of processes $\implies$ resource theory of genuine non-Markovianity

Thank you

References

1. F. Buscemi, *On complete positivity, Markovianity, and the quantum data-processing inequality, in the presence of initial system-environment correlations*. Physical Review Letters, vol. 113, 140502 (5pp), 2014.

2. F. Buscemi, R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, and M.N. Bera, *Information revival without backflow: non-causal explanations of non-Markovianity*. PRX Quantum, vol. 6, 020316 (12pp), 2025.