Squashed information backflows in non-Markovian quantum stochastic processes

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23 May 2023

Abstract

It is well known that convex combinations of Markov processes typically result in non-Markov ones. In this talk I will review some notions of (non-)Markovianity for quantum stochastic processes focusing in particular on a recent proposal to quantify information backflows after classical memories have been suitably squashed out. Such a "squashed" non-Markovianity, besides suggesting a notion of "genuine" or "causal" information revivals, is also able to resolve the problem of non-convexity, thus clarifying the role of non-Markovianity as a resource. The possibility of extending the same intuition to other non-convex resource theories is discussed. This is joint work with R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, and M.N. Bera. Preprint available as arXiv:2405.05326.

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Bridging Quantum Information and Mathematical Physics
Centre for Mathematical Sciences
Cambridge, 14-16 August 2024

References

- F.B.: *On complete positivity, Markovianity, and the quantum data-processing inequality, in the presence of initial system-environment correlations.* Physical Review Letters 113, 140502 (2014)

- F.B., R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, M.N. Bera: *Information revival without backflow: non-causal explanations of non-Markovianity.* Preprint arXiv:2405.05326

Reduced dynamics in the presence of initial system–environment correlations

The problem in a nutshell

textbooks usually begin with the factorization assumption, i.e., $\bullet_Q \otimes \gamma_E$: in this case, the reduced dynamics $\mathrm{Tr}_{E'}\left[U_{QE\to Q'E'}\ (\bullet_Q \otimes \gamma_E)\ U^\dagger_{QE\to Q'E'}\right]$ is always well defined, completely positive and trace-preserving

- 1994: Pechukas' PRL (what if we drop the factorization assumption?) and Alicki's comment on it
- 2004: Sudarshan's group (explicit constructions and examples)
- 2009: Shabani and Lidar's PRL (claim: quantum discord solves the problem)
- 2013: Brodutch et al's counterexample voiding the Shabani–Lidar PRL
- 2014: next slide

Preparable initial conditions

- initial set of possible system-environment states $\sigma_{QE} \ni \mathfrak{S}_{QE} \subseteq \mathcal{S}(\mathcal{H}_Q \otimes \mathcal{H}_E)$
- **requirement of preparability**: the set $\mathfrak{S}_{QE}$ is said to be *preparable* if and only if there exists an input system R and a CP linear map $\mathcal{P}: R \to QE$ such that $\mathfrak{S}_{QE}$ is the filter of $\mathcal{S}(\mathcal{H}_R)$ under $\mathcal{P}$, that is,

$$\mathfrak{S}_{QE} = \mathcal{P}(\mathcal{S}(\mathcal{H}_R)) := \left\{ \frac{\mathcal{P}(\varrho_R)}{\mathrm{Tr}[\mathcal{P}(\varrho_R)]} : \varrho_R \in \mathcal{S}(\mathcal{H}_R) \ \wedge \ \mathrm{Tr}[\mathcal{P}(\varrho_R)] > 0 \right\}$$

- **equivalence with steerability**: the set $\mathfrak{S}_{QE}$ is preparable if and only if it is *steerable*, i.e., if and only if there exists a reference system R and a tripartite density operator ω_{RQE} such that

$$\forall \sigma_{QE} \in \mathfrak{S}_{QE} \ , \ \exists \pi_R \geqslant 0 \ : \ \sigma_{QE} = \frac{\mathrm{Tr}_R[\omega_{RQE}\ (\pi_R \otimes \mathbb{1}_{QE})]}{\mathrm{Tr}[\omega_{RQE}\ (\pi_R \otimes \mathbb{1}_{QE})]}$$

Result (PRL, 2014)

Fact

Let the set $\mathfrak{S}_{QE}$ be a preparable/steerable set of initial system-environment conditions. The following are equivalent:

- $\mathfrak{S}_{QE}$ is **CPTP reducible**: for any interaction $U_{QE \to Q'E'}$, there exists a corresponding CPTP linear map $\mathcal{E}_{Q \to Q'}$ such that

$$\mathrm{Tr}_{E'}\left[U \sigma_{QE} U^\dagger\right] = \mathcal{E} \circ \mathrm{Tr}_E[\sigma_{QE}] \ , \quad \forall \sigma_{QE} \in \mathfrak{S}_{QE}$$

- $\mathfrak{S}_{QE}$ is **Markov-steerable**: there exists a tripartite state ω_{RQE} with $I(R;E|Q) = 0$, such that $\mathfrak{S}_{QE}$ is steerable from ω_{RQE}

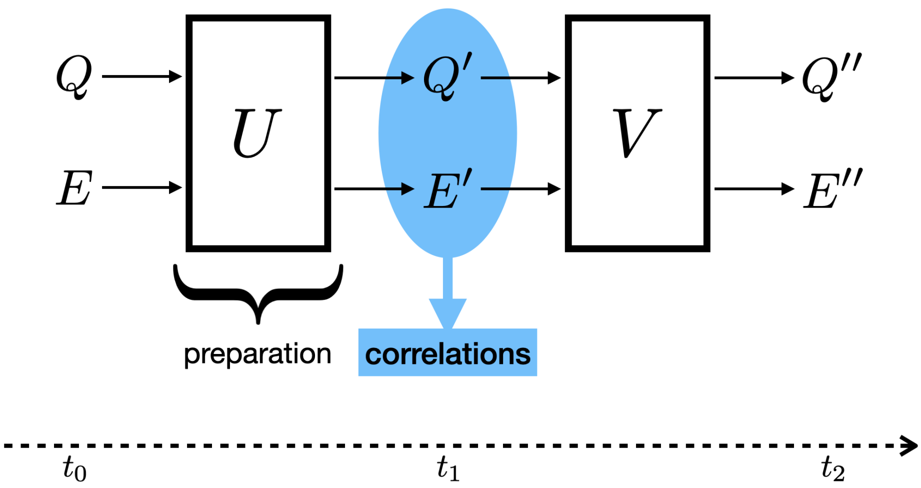
all known examples fall within the scope of the above theorem, which also makes it much easier to verify the CPTP reducibility condition, but many more can be constructed.

What happens when the set of initial assignments is not CPTP-reducible?

Information revival 1/2

let us consider the first interaction step as the "preparation" procedure:

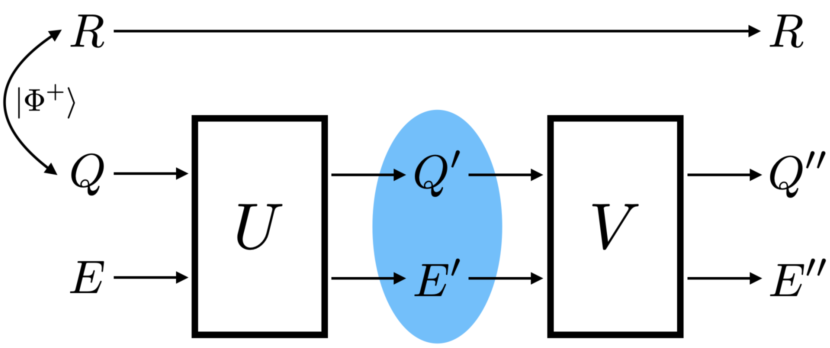
$$\mathcal{S}(\mathcal{H}_Q) \otimes \gamma_E \xrightarrow{U_{QE}:t_0 \to t_1} \mathfrak{S}_{Q'E'} \xrightarrow{V_{Q'E'}:t_1 \to t_2} \mathfrak{S}_{Q''E''}$$



Information revival 2/2

for convenience, we introduce a reference system $\mathcal{H}_R \cong \mathcal{H}_Q$ and a maxent state $|\Phi^+\rangle_{RQ}$

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow{t_0 \to t_1} \underbrace{U_{QE}(\Phi^+_{RQ} \otimes \gamma_E)U^\dagger_{QE}}_{\equiv \sigma_{RQ'E'}} \xrightarrow{t_1 \to t_2} \underbrace{V_{Q'E'}\sigma_{RQ'E'}V^\dagger_{Q'E'}}_{\equiv \tau_{RQ''E''}}$$

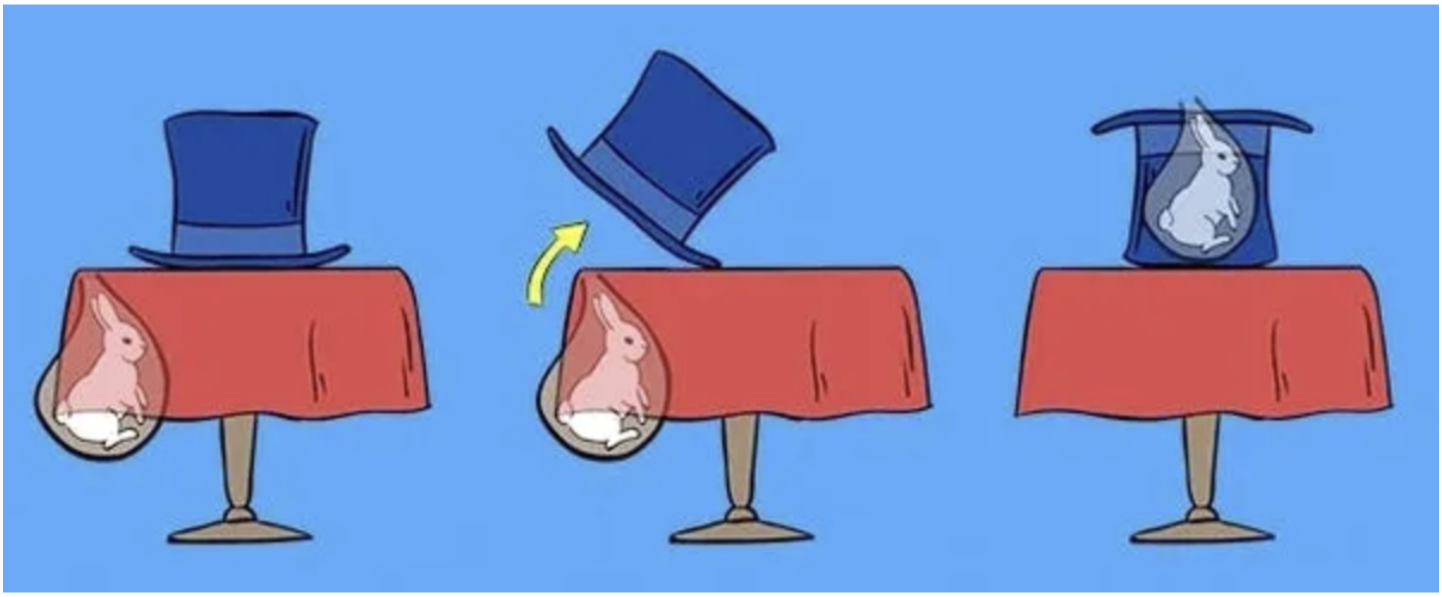


if $I(R;E'|Q') > 0$, a revival, i.e., $I(R;Q'') > I(R;Q')$, may occur

By looking at the system alone, a revival amounts to a violation of locality!

As such, it needs an explanation.

Explaining revivals



Explaining revivals

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow[t_0 \to t_1]{U_{QE}} \sigma_{RQ'E'} \xrightarrow[t_1 \to t_2]{V_{Q'E'}} \tau_{RQ''E''} \tag{1}$$

suppose that a revival happens between t_1 and t_2, i.e., $I(R;Q'') > I(R;Q')$

Explanation: compatibly with (1), keep adding parts of the universe to Q', until the revival disappears, i.e., $I(R;Q'\cdots) \geqslant I(R;Q''\cdots)$

Obvious explanation: just add the environment itself! Indeed, $I(R;Q'E') \geqslant I(R;Q''E'')$

$\Longrightarrow$ **information backflow**

Statement no. 1

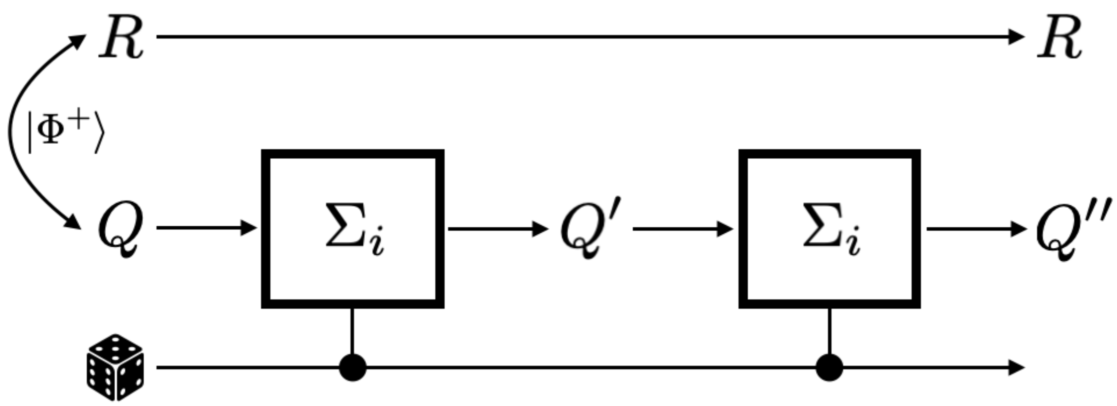
If $I(R;E'|Q')>0$, information revivals are possible.

Any revival can be explained as a backflow.

But is a backflow always necessary?

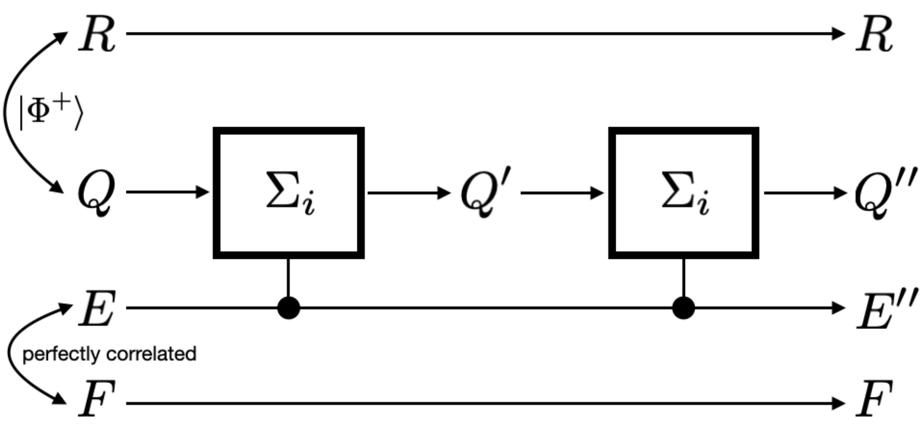
A motivating example 1/2

$\mathcal{H}_R \cong \mathcal{H}_Q \cong \mathbb{C}^2\,, \qquad \gamma_E = \frac{1}{4}\mathbb{1}\,, \qquad \Sigma^i \in \{\mathbb{1}, X, Y, Z\}$



$$I(R;Q)=2 \xrightarrow{t_0\to t_1} I(R;Q')=0 \xrightarrow{t_1\to t_2} I(R;Q'')=2$$

A motivating example 2/2



$$I(R;QF) = 2 \xrightarrow{t_0 \to t_1} I(R;Q'F) = 2 \xrightarrow{t_1 \to t_2} I(R;Q''F) = 2$$

$\implies$ F provides an explanation, even though it never interacts with Q and is causally separated from it at all times!

$\implies$ there **cannot** be any "backflow" from F into Q

Non-causal explanations

- start from

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow{U_{QE}} \sigma_{RQ'E'} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''}$$

and take an extension of γ_E using an ancillary system F

$$\Phi^+_{RQ} \otimes \gamma_{EF} \xrightarrow{U_{QE}} \sigma_{RQ'E'F} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''F}$$

- the extension F never interacts with the system: it may reside in a space-like separated (thus, causally separated) region when the first interaction between Q and E takes place
- and yet, F could explain the information revival, that is, $I(R;Q') < I(R;Q'')$ but $I(R;Q'F) \geqslant I(R;Q''F)$
- in this case, the extension F provides a **non-causal explanation**: the information revival **can be** explained without the need for any backflow

Statement 2

A backflow is not always necessary to explain information revivals.

So, when is a backflow absolutely necessary? And when instead do we never need one?

Sufficient condition for a causal backflow

- suppose that there is a revival, i.e., $I(R;Q') < I(R;Q'')$
- a non-causal explanation does not exists if and only if

$$\forall F: \text{ non-causal extensions }, \quad I(R;Q'F) < I(R;Q''F)$$

- a sufficient condition for the above is

$$\sup_F I(R;Q'|F) < \inf_F I(R;Q''|F)$$

- in turn, the above holds if

$$H(Q')_\sigma < E_{sq}(\tau_{RQ''})$$

Statement 3

If $H(Q')_\sigma < E_{sq}(\tau_{RQ''})$, then the revival requires a backflow, regardless of the interaction model.

Are there processes that never require a backflow?

Non-causal correlations

$$\Phi^+_{RQ} \otimes \gamma_{EF} \xrightarrow{U_{QE}} \sigma_{RQ'E'F} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''F}$$

$$I(R;Q'F) \geqslant I(R;Q''F) \iff I(R;E''|Q''F) \geqslant I(R;E'|Q'F)$$

non-causal correlations: if there exists a causally separated extension F such that $I(R;E'|Q'F) = 0$, the system-environment correlations present at time $t = t_1$ are called non-causal

Statement 4

If there exists F such that $I(R;E'|Q'F)$, only non-causal revival are possible:
a backflow is never required.

Added bonus: closure under convex mixtures

take the mixture of two processes

$$p\left\{\Phi^+_{RQ}\otimes\gamma^{(a)}_E\to\sigma^{(a)}_{RQ'E'}\to\tau^{(a)}_{RQ''E''}\right\}+(1-p)\left\{\Phi^+_{RQ}\otimes\gamma^{(b)}_E\to\sigma^{(b)}_{RQ'E'}\to\tau^{(b)}_{RQ''E''}\right\}$$

even if both $\sigma^{(a)}_{RQ'E'}$ and $\sigma^{(b)}_{RQ'E'}$ only contain inert correlations, their mixture could allow revivals, i.e.,

$$I(R;E'|Q')_a=0\ \wedge\ I(R;E'|Q')_b=0\quad\not\Longrightarrow\quad I(R;E'|Q')_{pa+(1-p)b}=0$$

instead, any convex mixture of non-causal correlations is automatically non-causal

we can construct a convex resource theory of **genuine** (causal) **non-Markovian backflows**

Conclusion

Today's take-home ideas

- in open quantum systems dynamics, the separation is not only "revival occurs" (non-Markov) VS "revival does not occur" (Markov)
- within revivals, we can further distinguish between "non-causal revivals" VS "genuine backflows"
- if $\exists F$ such that $I(R;E'|Q'F)=0$, then only non-causal revivals
- if $H(Q') < E_{sq}(R;Q'')$, then genuine backflow
- such "genuine non-Markovianity" is well-behaved under convex mixtures of processes $\Longrightarrow$ resource theory of genuine non-Markovianity

Thank you

References

1. F. Buscemi, *On complete positivity, Markovianity, and the quantum data-processing inequality, in the presence of initial system-environment correlations*. Physical Review Letters, vol. 113, 140502 (5pp), 2014.

2. F. Buscemi, R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, and M.N. Bera, *Information revival without backflow: non-causal explanations of non-Markovianity*. Preprint arXiv:2405.05326, 2024.