Time-Like Correlations and Retrodiction in Quantum Theory

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Abstract

Extending classical multitime processes to quantum dynamics presents significant challenges, particularly due to the "no information without perturbation" principle. Instead of hindering progress, these challenges have driven the development of innovative and sometimes unconventional methods that question traditional interpretations of quantum mechanics. In this talk I will review some of the results we have obtained in the last decade that shed light on the role of time-like correlations in quantum theory. This is work done in collaboration with: Giulio Chiribella, Michele Dall'Arno, James Fullwood, and Arthur Parzygnat.

Time-Like Correlations ~~and Retrodiction~~ in Quantum Theory

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References

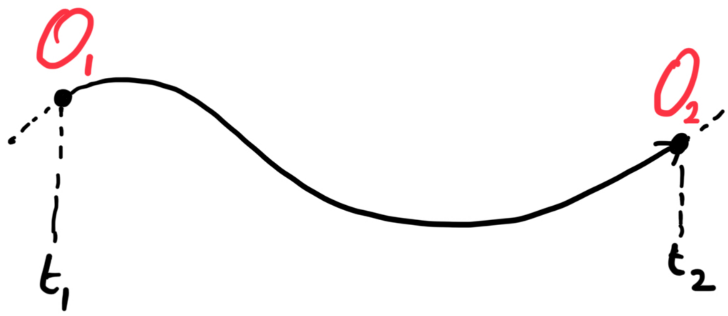
Presentation based on:

- *Direct observation of any two-point quantum correlation function*. Preprint arXiv:1312.4240
- *Universal optimal quantum correlator*. International Journal of Quantum Information 12, 1560002 (2014)
- *Virtual quantum broadcasting*. Physical Review Letters 132, 110203 (2024)

Collaborators on this journey: G. Chiribella, M. Dall'Arno, J. Fullwood, and A. J. Parzygnat

The problem with correlations in time

Consider a single system stochastically evolving from state x at time t_1 to state y at time $t_2 > t_1$, and two observables, $O_1 = O_1(x)$ and $O_2 = O_2(y)$, one for each time.



classical case: $\langle O_1 O_2 \rangle_{t_1 \to t_2} = \sum_{x,y} O_1(x) O_2(y) \, \mathrm{Pr}\{x, t_1; y, t_2\}$

quantum case: observation is intervention and time-correlations depend on how the first observation is performed

Is there a canonical form for quantum time-correlations?

How you do it classically

- fix orthonormal bases $\{|x\rangle\}_x$ and $\{|y\rangle\}_y$
- choose an initial state $\mathbf{p} = \sum_x p(x)|x\rangle\langle x|$
- apply the ideal *classical* correlator $\mathcal{C}(\mathbf{p}) = \sum_x p(x)|x\rangle\langle x|_1 \otimes |x\rangle\langle x|_2$
- let system 2 evolve:

$$\sum_{x,y} \underbrace{p(x)N(y|x)}_{\Pr\{x,t_1;y,t_2\}} |x\rangle\langle x|_1 \otimes |y\rangle\langle y|_2$$

- measure (compute) O_1 on system 1 and O_2 on system 2:

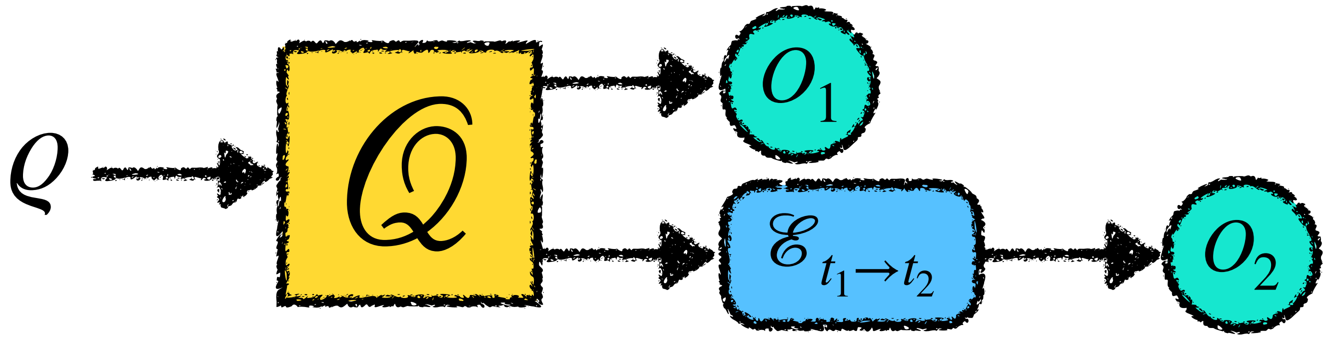
$$\begin{aligned}\langle O_1 O_2\rangle_{t_1\to t_2} &= \sum_{x,y} p(x)N(y|x)O_1(x)O_2(y) \\ &= \sum_x p(x)O_1(x) \underbrace{\sum_y N(y|x)O_2(y)}_{\tilde{O}_2(x)}\end{aligned}$$

The crucial ingredient is the ideal correlator.

What is its quantum analog?

Operational picture

If we had an "ideal quantum correlator" $\mathcal{Q}$, we could do the same:



Properties of $\mathcal{Q}$

Requirements for the ideal *quantum* correlator:

- linearity: $\mathcal{Q}(c_1 X_1 + c_2 X_2) = c_1 \mathcal{Q}(X_1) + c_2 \mathcal{Q}(X_2)$
- consistency with the classical correlator: $(\Delta \otimes \Delta)\ \mathcal{Q}(\Delta(\cdot)) = \mathcal{C}(\Delta(\cdot))$, where $\Delta(\cdot) = \sum_x |x\rangle\langle x| \cdot |x\rangle\langle x|$
- symmetry: $\texttt{SWAP}\ \mathcal{Q}(\cdot)\ \texttt{SWAP} = \mathcal{Q}(\cdot)$
- universal covariance: for any unitary U, $\mathcal{Q}(U \cdot U^\dagger) = (U \otimes U)\mathcal{Q}(\cdot)(U^\dagger \otimes U^\dagger)$

Remark. Universal covariance guarantees that classical consistency is basis-independent.

Uniqueness result

The ideal quantum correlator (PRL, 2024)

The four requirements of linearity, classical consistency, symmetry, and universal covariance single out a unique map given by

$$\mathcal{Q}(\varrho) := \frac{\mathtt{SWAP}(\varrho \otimes \mathbb{1}) + (\varrho \otimes \mathbb{1})\mathtt{SWAP}}{2} ,$$

where $\mathtt{SWAP}(|\psi\rangle \otimes |\phi\rangle) = |\phi\rangle \otimes |\psi\rangle$ for all vectors $|\psi\rangle$ and $|\phi\rangle$.

For example, if $\varrho = |\psi\rangle\langle\psi|$, then

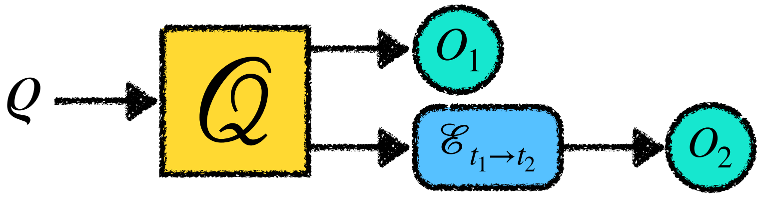
$$\mathcal{Q}(\varrho) = \frac{1}{2} \sum_x \Big(|x\rangle\langle\psi| \otimes |\psi\rangle\langle x| + \mathrm{cc} \Big) .$$

Consequences

As a *result*, $\mathcal{Q}$ satisfies:

- hermite-preservation (HP): $X = X^\dagger \implies \mathcal{Q}(X) = [\mathcal{Q}(X)]^\dagger$
- trace-preservation (TP): $\mathrm{Tr}[\mathcal{Q}(X)] = \mathrm{Tr}[X]$
- ideal broadcasting: $\mathrm{Tr}_1[\mathcal{Q}(\varrho)] = \mathrm{Tr}_2[\mathcal{Q}(\varrho)] = \varrho$, for all ϱ

Use of $\mathcal{Q}$ to generate quantum time-correlations

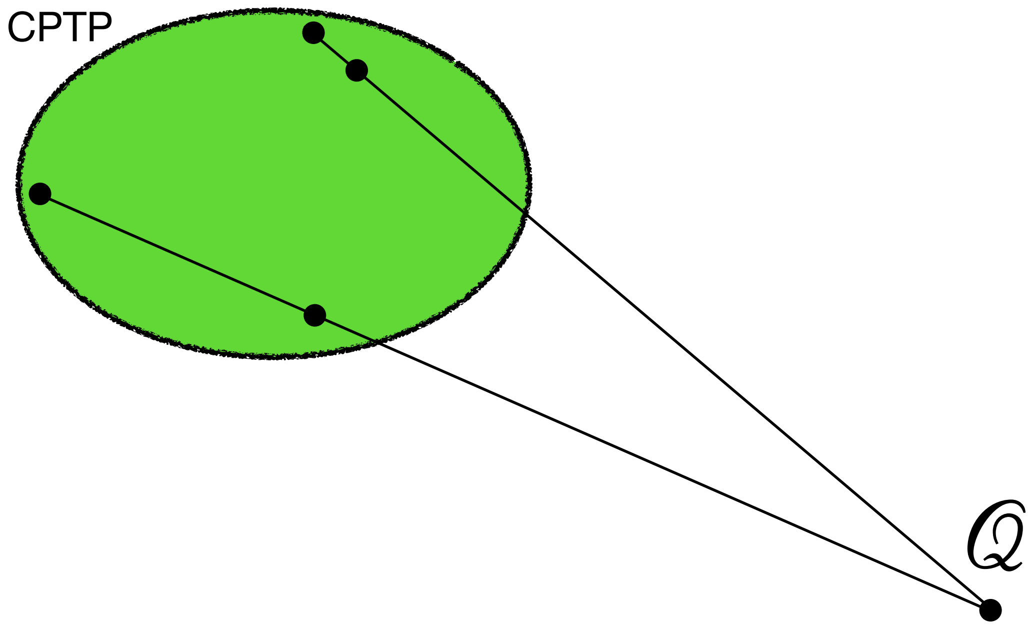


$$\langle O_1 O_2\rangle^{Q}_{t_1\to t_2} := \mathrm{Tr}[(\mathsf{id}\otimes\mathcal{E}_{t_1\to t_2})\mathcal{Q}(\varrho)\ (O_1\otimes O_2)] = \mathrm{Tr}\left[\varrho\ \frac{O_1\overbrace{\mathcal{E}^\dagger(O_2)}^{\tilde{O}_2}+\mathcal{E}^\dagger(O_2)O_1}{2}\right]$$

The above corresponds to the symmetric part of the Dirac–Kirkwood quasi-probability distribution, which thus gets a new operational interpretation as the unique "canonical" analog of classical broadcasting.

However, $\mathcal{Q}$ is not a valid quantum channel (it is not positive)

Optimal physical approximation



Optimal physical approximation

arXiv:1312.4240; PRL (2024)

The ideal quantum correlator satisfies

$$\arg\min_{\mathcal{E}:\text{CPTP}} \|\mathcal{Q}-\mathcal{E}\|_\diamond = \{\mathcal{S}\}$$

where $\mathcal{S}$ is the Bužek–Hillery–Werner symmetric optimal one-to-two universal quantum cloning map given by

$$\mathcal{S}(\cdot) := \frac{2}{d+1}\Pi_{\text{symm}}(\varrho \otimes \mathbb{1})\Pi_{\text{symm}} \ ,$$

where $\Pi_{\text{symm}} := \frac{1}{2}(\mathbb{1} + \texttt{SWAP})$ is the projector on the symmetric subspace of $\mathbb{C}^d \otimes \mathbb{C}^d$.

Remark. A connection between Dirac–Kirkwood distribution and cloning is also noted in [Hofmann, PRL (2012)], but without any discussion about optimality or uniqueness.

Optimal simulation protocol

arXiv:1312.4240

The unique optimal* statistical decomposition for the ideal quantum correlator is given by

$$\mathcal{Q}(\cdot) = \frac{d+1}{2}\mathcal{S}(\cdot) - \frac{d-1}{2}\mathcal{A}(\cdot) \ ,$$

where

$$\mathcal{S}(\cdot) := \frac{2}{d+1}\Pi_{\mathsf{symm}}(\varrho \otimes \mathbb{1})\Pi_{\mathsf{symm}} \qquad \mathcal{A}(\cdot) := \frac{2}{d-1}\Pi_{\mathsf{anti}}(\varrho \otimes \mathbb{1})\Pi_{\mathsf{anti}} \ ,$$

where $\Pi_{\mathsf{symm}} := \frac{1}{2}(\mathbb{1} + \mathtt{SWAP})$ and $\Pi_{\mathsf{anti}} := \frac{1}{2}(\mathbb{1} - \mathtt{SWAP})$ are, respectively, the projector on the symmetric and on the antisymmetric subspaces of $\mathbb{C}^d \otimes \mathbb{C}^d$.

* optimal because $\|\mathcal{Q}\|_\diamond = \left\|\frac{d+1}{2}\mathcal{S} - \frac{d-1}{2}\mathcal{A}\right\|_\diamond = \frac{d+1}{2}\|\mathcal{S}\|_\diamond + \frac{d-1}{2}\|\mathcal{A}\|_\diamond = d.$

Interferometric scheme (arXiv:1312.4240)

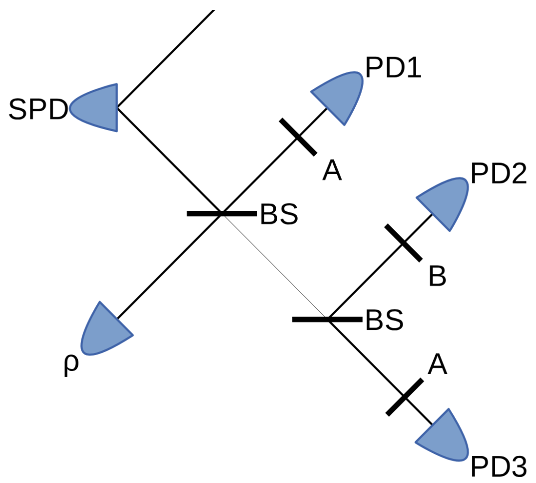


FIG. 4. Experimental proposal for the probabilistic implementation of the real part of the universal optimal two-point correlator for qubits. Thin lines represent optical qubits encoded in the polarization of photons, BS is a 50/50 beamsplitter, SPD is a source of maximally entangled photons through spontaneous parametric downconversion, ρ is the input state fed into the universal optimal two-point correlator, A and B are the phase shifters implementing the corresponding observables, PD1, PD2, and PD3 are photodetectors. A coincidence occurs between PD2 and PD3 (resp., PD1 and PD2) with probability 3/16 (resp., 1/16), in this case optimal universal symmetric (resp., antisymmetric) cloning has been per-

Conclusion

Take-home messages

- there is a unique, canonical, ideal quantum correlator
- no *ad hockeries* or arguments from authority, but only four natural requirements (linearity, classical consistency, symmetry, unitary covariance)
- the ideal quantum correlator generates a canonical representation of time correlations (a canonical state-over-time)
- the ideal quantum correlator is not positive: another manifestation of the negative signature of time?
- the optimal universal quantum cloning is the unique optimal physical approximation

Time to venture into new territories of quantum theory!

References

1. F. Buscemi, M. Dall'Arno, M. Ozawa, and V. Vedral: *Direct observation of any two-point quantum correlation function.* Preprint arXiv:1312.4240

2. F. Buscemi, M. Dall'Arno, M. Ozawa, and V. Vedral: *Universal optimal quantum correlator.* International Journal of Quantum Information, vol. 12, 1560002 (2014)

3. A.J. Parzygnat, J. Fullwood, F. Buscemi, and G. Chiribella: *Virtual quantum broadcasting.* Physical Review Letters, vol. 132, 110203 (2024)