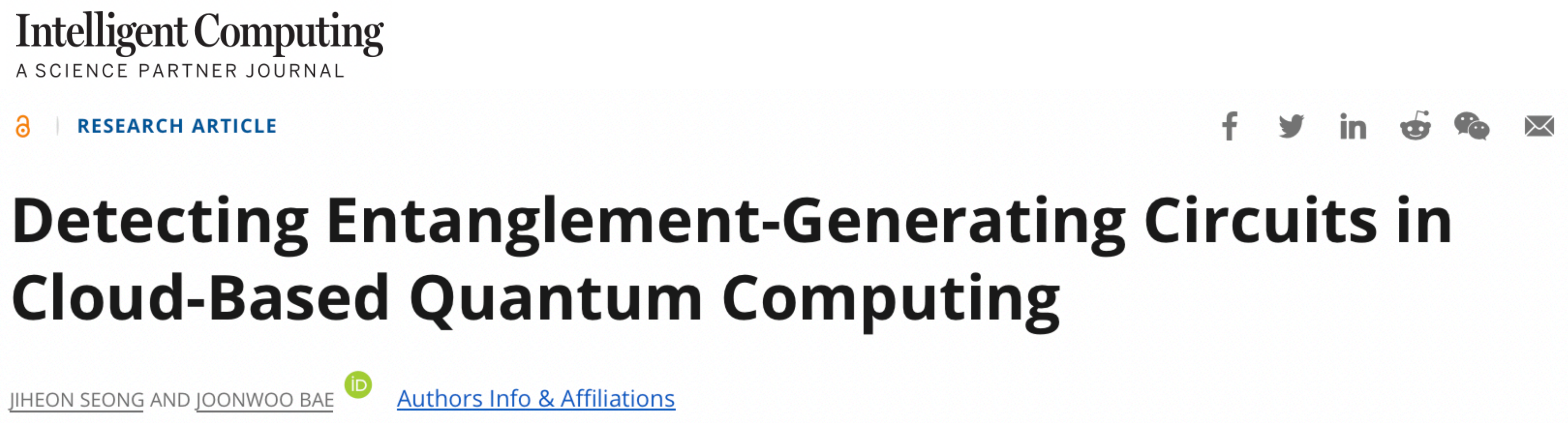
# Detecting Entanglement-Generating Circuits in Cloud-Based Quantum Computing

## Talk @ Nagoya-KAIST GEnKO 2023 Workshop

### on Quantum Entanglement and Open Quantum Systems

Jiheon Seong and Joonwoo Bae, **QIT@ KAIST**



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RESEARCH ARTICLE

Detecting Entanglement-Generating Circuits in Cloud-Based Quantum Computing

JIHEON SEONG AND JOONWOO BAE   Authors Info & Affiliations

- Jiheon Seong, Joonwoo Bae. Detecting Entanglement-Generating Circuits in Cloud-Based Quantum Computing. Intell Comput. 2023;2:0051.

# In This Talk ...

- **Quantum Entanglement** : a general and key resource that leads to quantum advantages

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  : EWs are a well-established tool for detecting entangled states without full knowledge gained via quantum tomography

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| **Preparation** | | |
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Quantum State

$$\rho$$

Quantum State

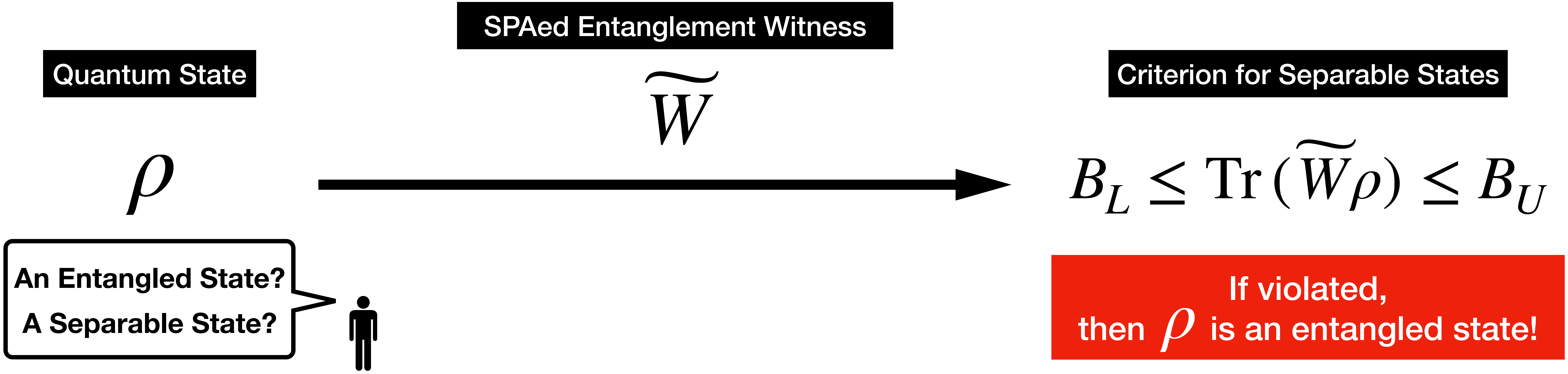
$$\rho$$

SPAed Entanglement Witness

$$\widetilde{W}$$

Quantum State

$$\rho$$



Quantum State
$\rho$
An Entangled State?
A Separable State?
SPAed Entanglement Witness
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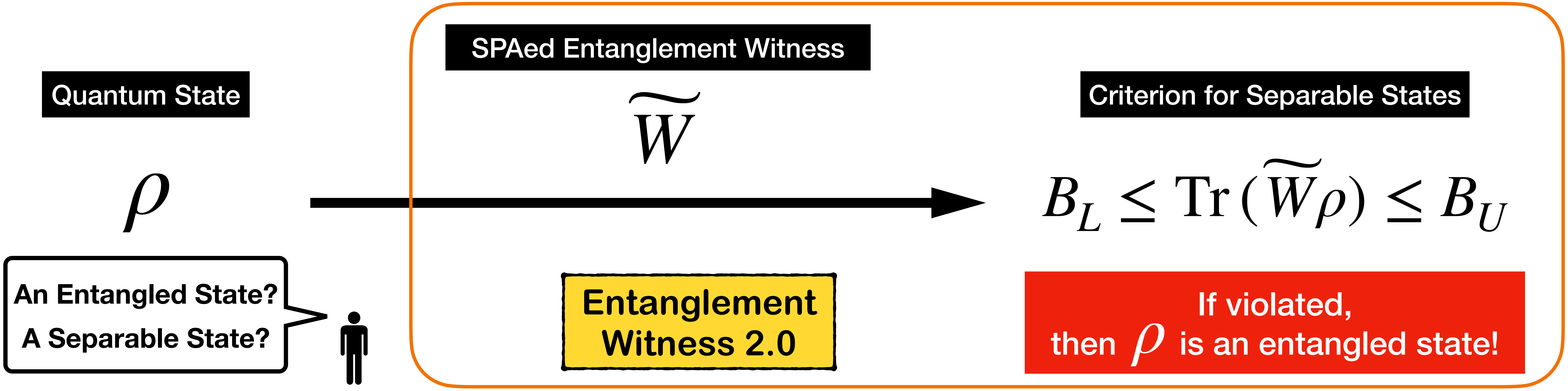
Criterion for Separable States
$B_L \leq \mathrm{Tr}(\widetilde{W}\rho) \leq B_U$
If violated,
then $\rho$ is an entangled state!



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A Separable State?

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Let's take a closer look at …

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**Quantum State**

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A Separable State?

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**Let's take a closer look at …**

$\widetilde{W}$

Hermitian: $\widetilde{W} = \widetilde{W}^{\dagger}$

Positive: $\widetilde{W} \geq 0$

Unit-Trace: $\mathrm{Tr}\,(\widetilde{W}) = 1$

**SPAed Entanglement Witness**

**Quantum State**

$$\rho \longrightarrow \widetilde{W}$$

**Criterion for Separable States**

$$B_L \leq \mathrm{Tr}\,(\widetilde{W}\rho) \leq B_U$$

An Entangled State?

A Separable State?

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If violated,
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Let’s take a closer look at …

$D = \mathrm{Tr}\, I \qquad 0 \leq p < 1,\ q > 1$

$W^{(\pm)}$ : Entanglement Witnesses

Quantum State

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Hermitian: $\widetilde{W} = \widetilde{W}^{\dagger}$

Positive: $\widetilde{W} \geq 0$

Unit-Trace: $\mathrm{Tr}\,(\widetilde{W}) = 1$

$$\widetilde{W} = (1-p)W^{(+)} + p\frac{I}{D}$$

$$\widetilde{W} = (1-q)W^{(-)} + q\frac{I}{D}$$

**SPAed Entanglement Witness**

$$\widetilde{W}$$

**Quantum State**

$$\rho$$

**Criterion for Separable States**

$$B_L \leq \mathrm{Tr}\,(\widetilde{W}\rho) \leq B_U$$

An Entangled State?
A Separable State?

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**If violated, then $\rho$ is an entangled state!**

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**Let's take a closer look at ...**

$D = \mathrm{Tr}\, I \qquad 0 \leq p < 1,\ q > 1$

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For any separable states $\sigma$,

$$\mathrm{Tr}\,(W^{(\pm)}\sigma) \geq 0$$

For some entangled states $\rho^{(\pm)}$,

$$\mathrm{Tr}\,(W^{(+)}\rho^{(+)}) < 0$$

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**Quantum State**

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An Entangled State?
A Separable State?

**SPAed Entanglement Witness**

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**Let's take a closer look at ...**

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**Structural Physical Approximation**

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Can be implemented in a quantum circuit

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$$\widetilde{W} = (1-p)W^{(+)} + p\frac{I}{D}$$

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The efficiency of EW is doubled

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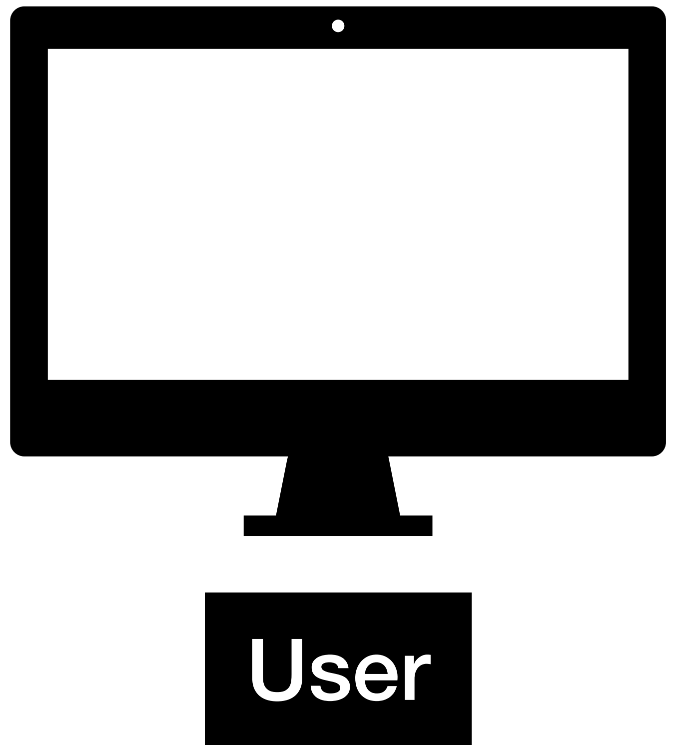
# Scenario of a cloud-based quantum computing service



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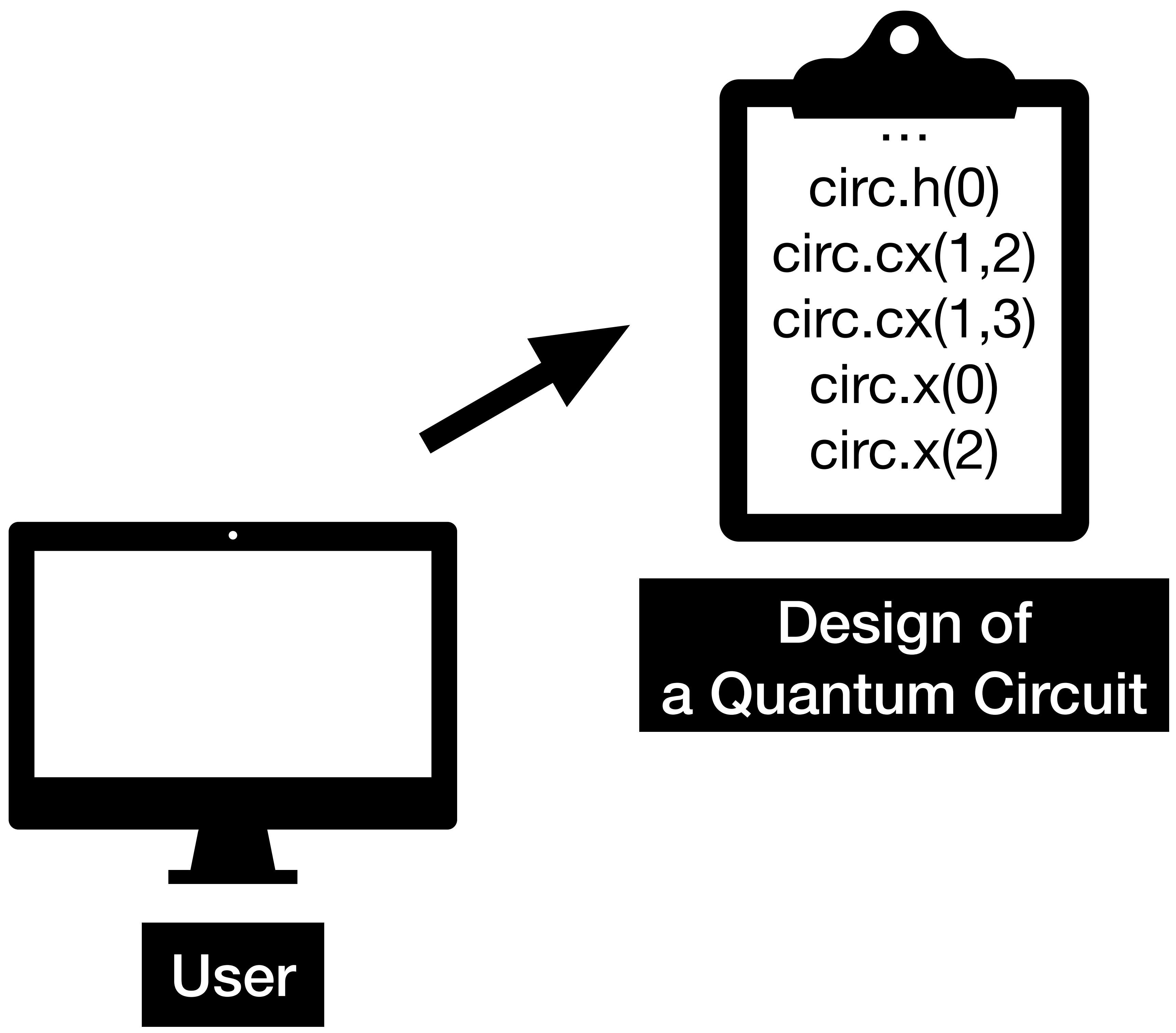

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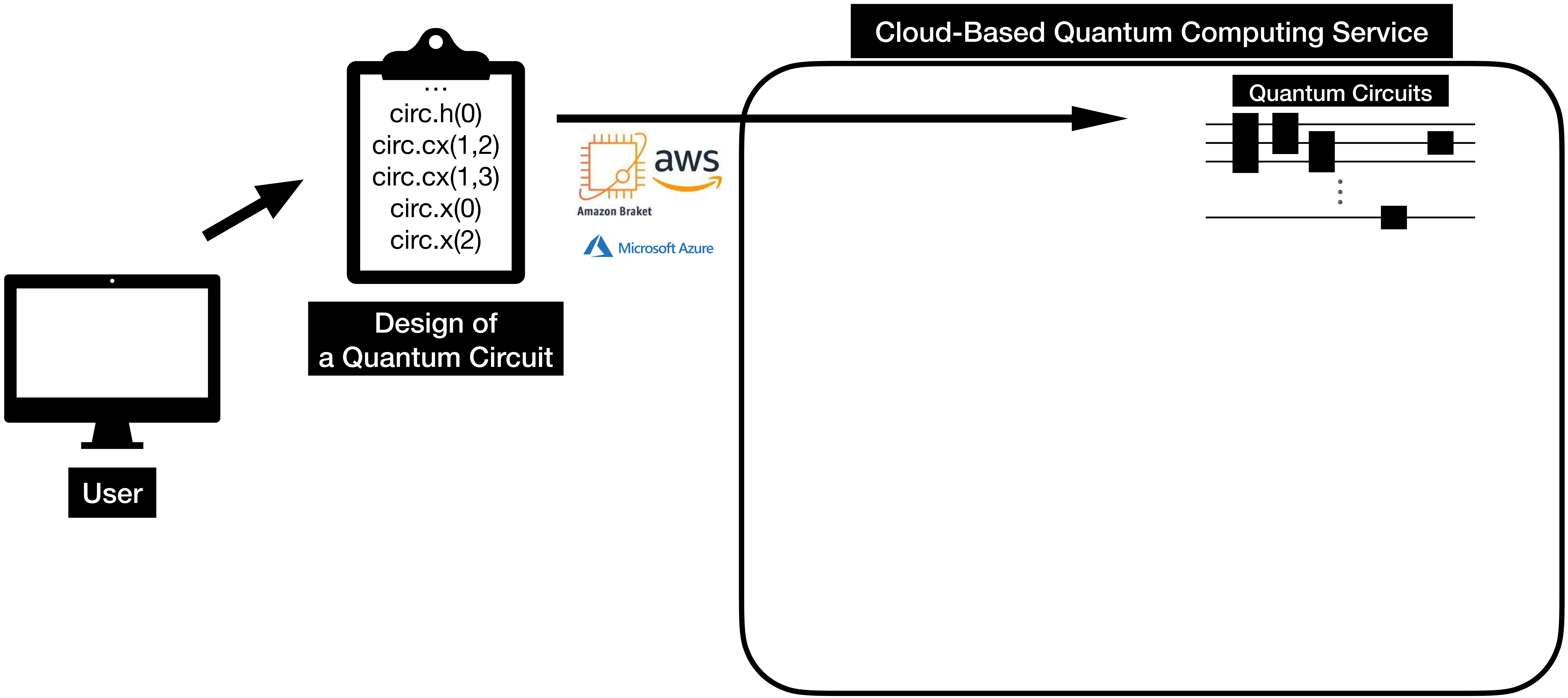

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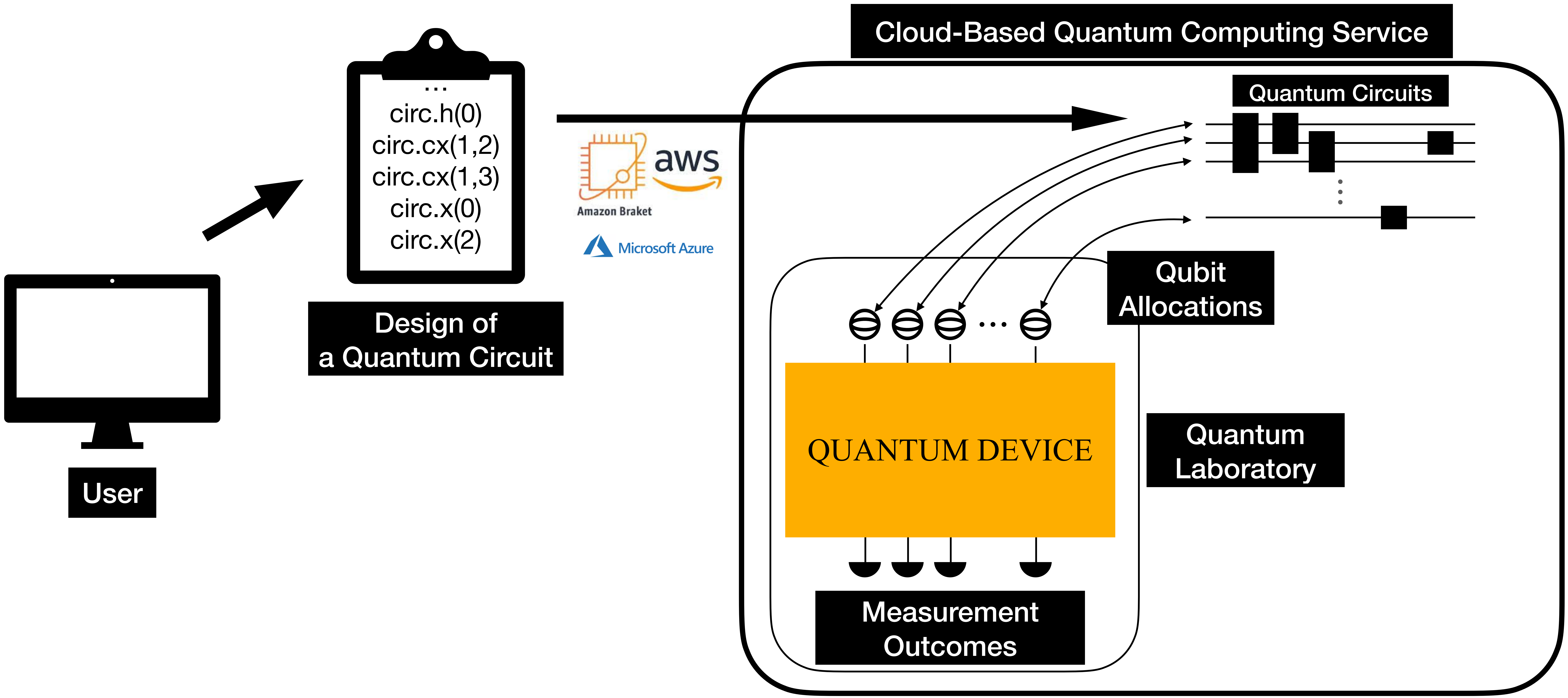

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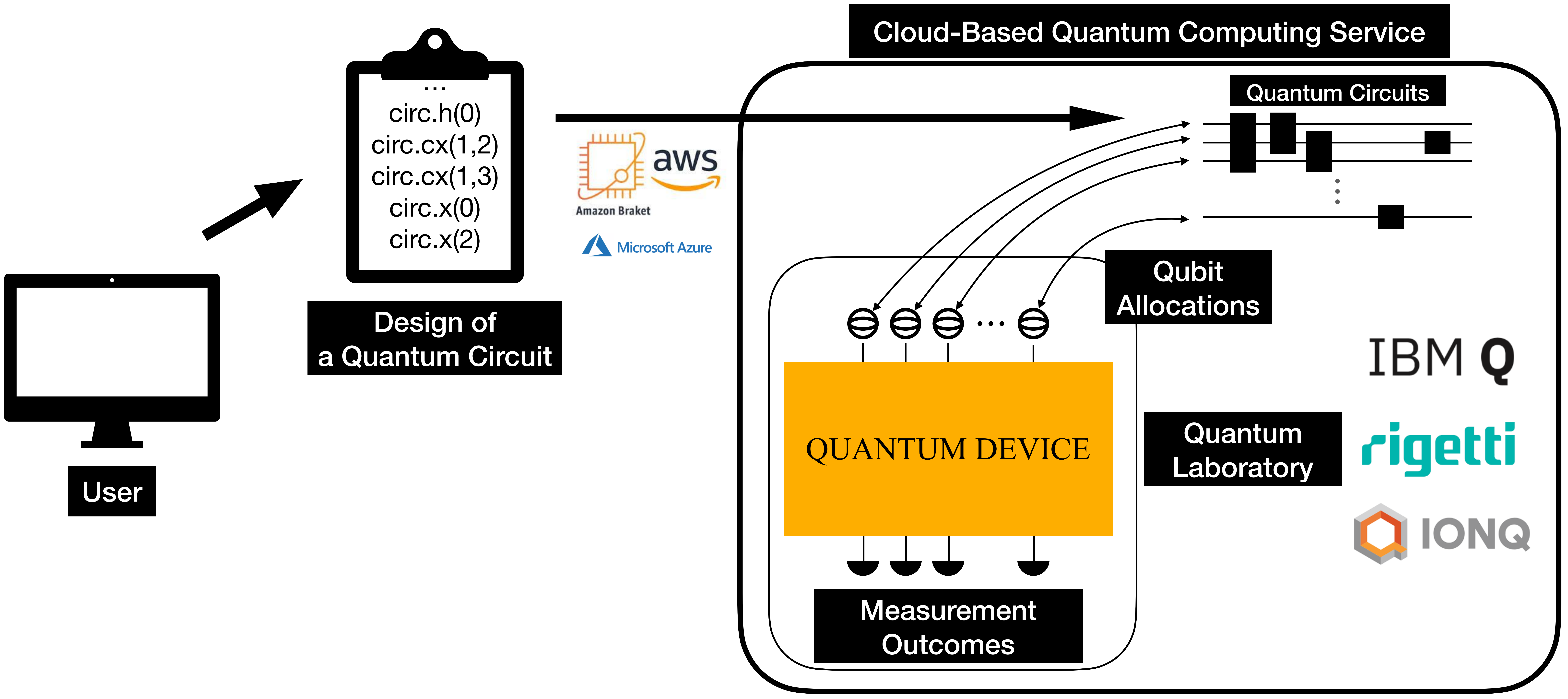

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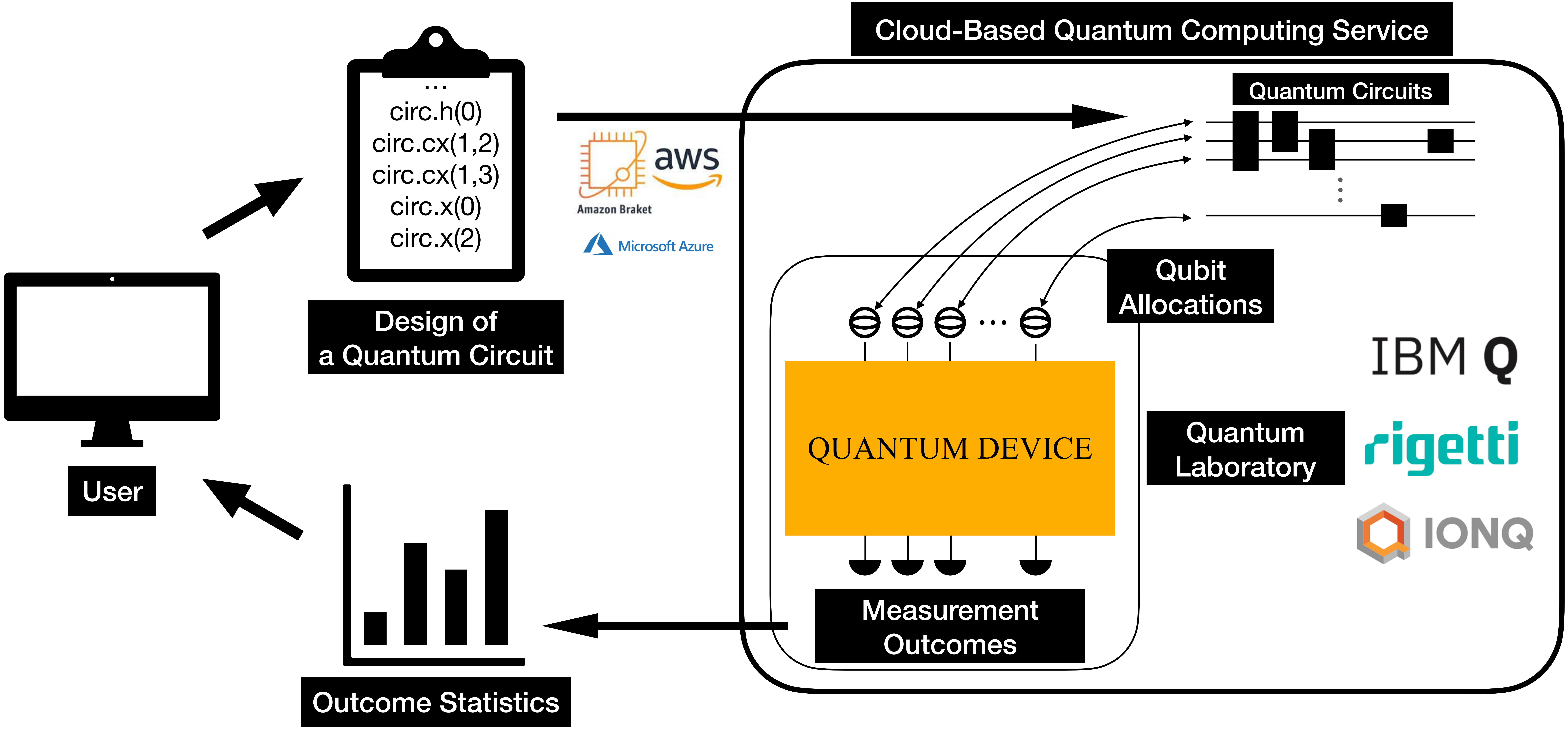

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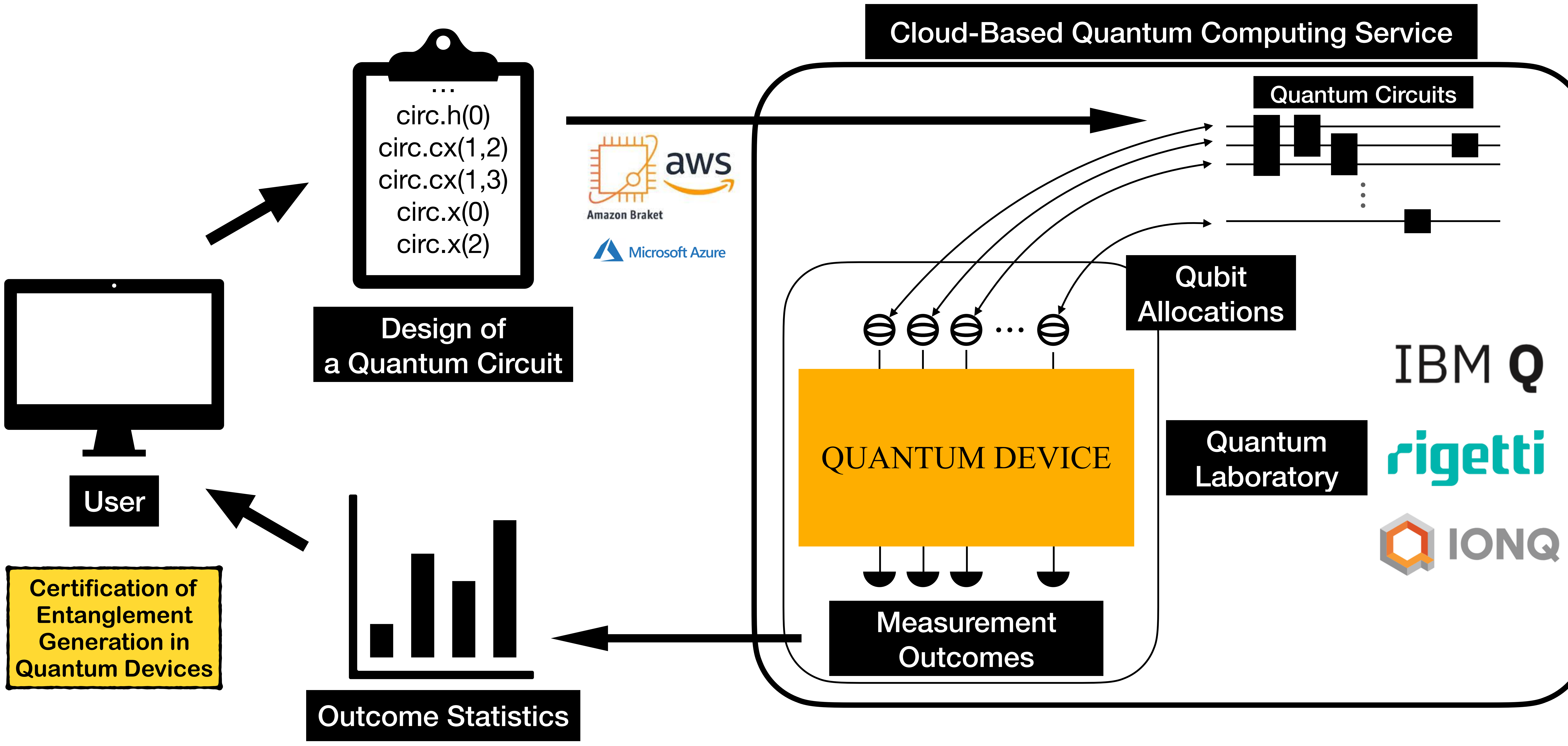

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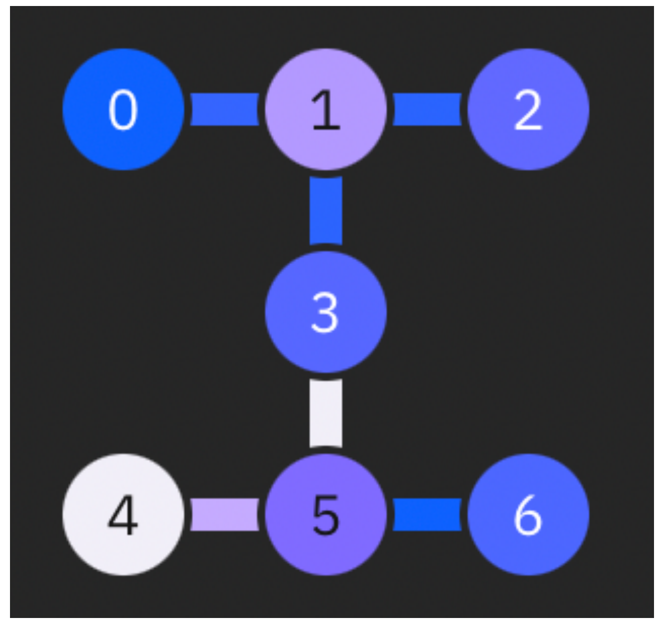
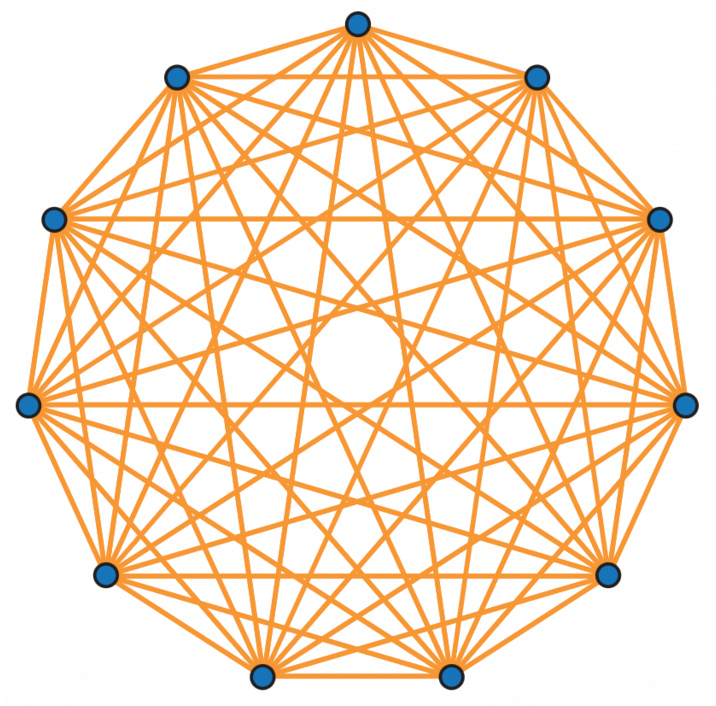
Scenario Certify generation of entanglement in such a quantum computer as (below)

with EW $W_{\mathrm{GHZ}_3} = 2I - (X_1X_2X_3 + Z_1Z_2 + Z_2Z_3)$

IBM Quantum ibm_nairobi




IonQ — Harmony



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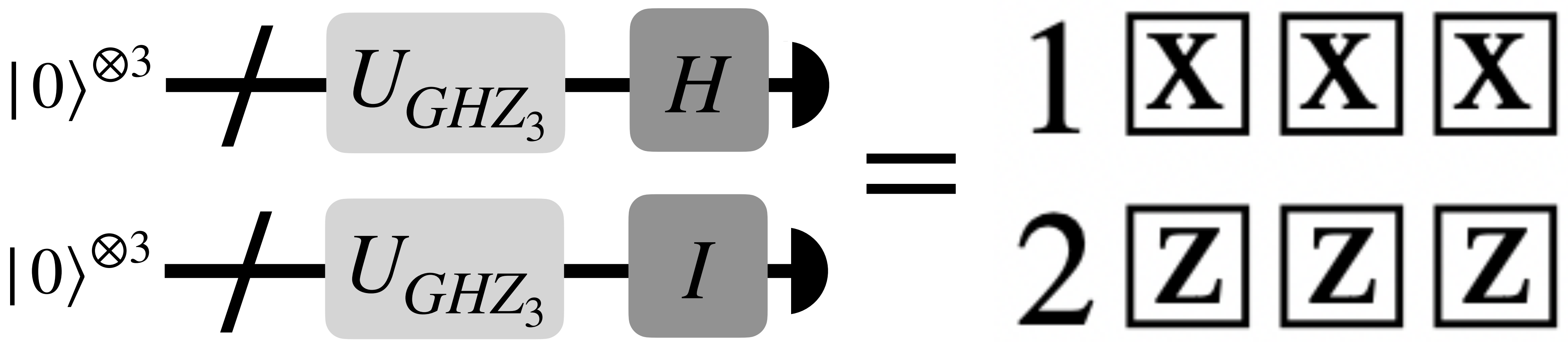


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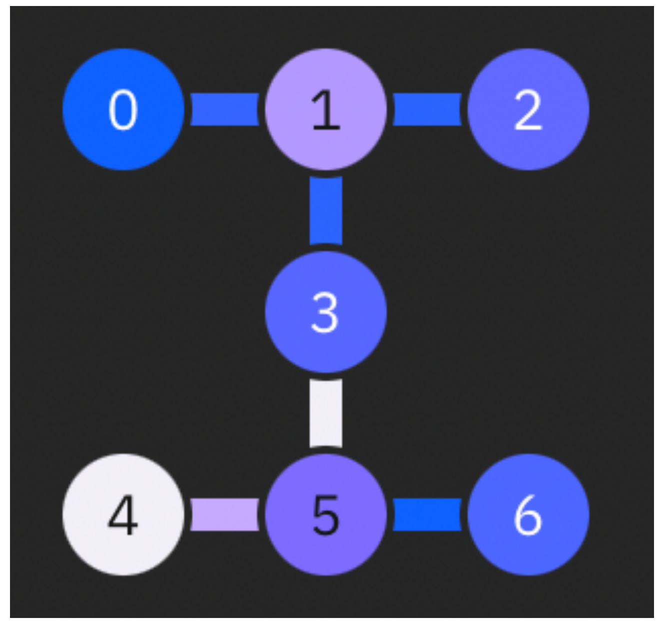


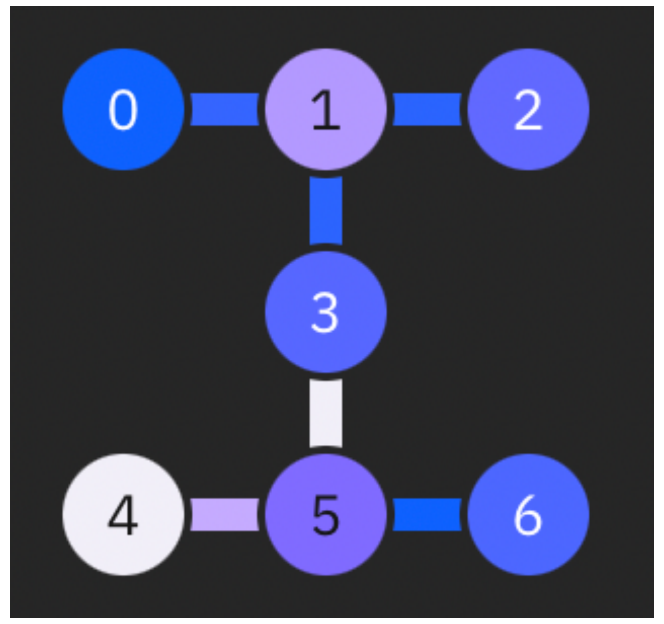


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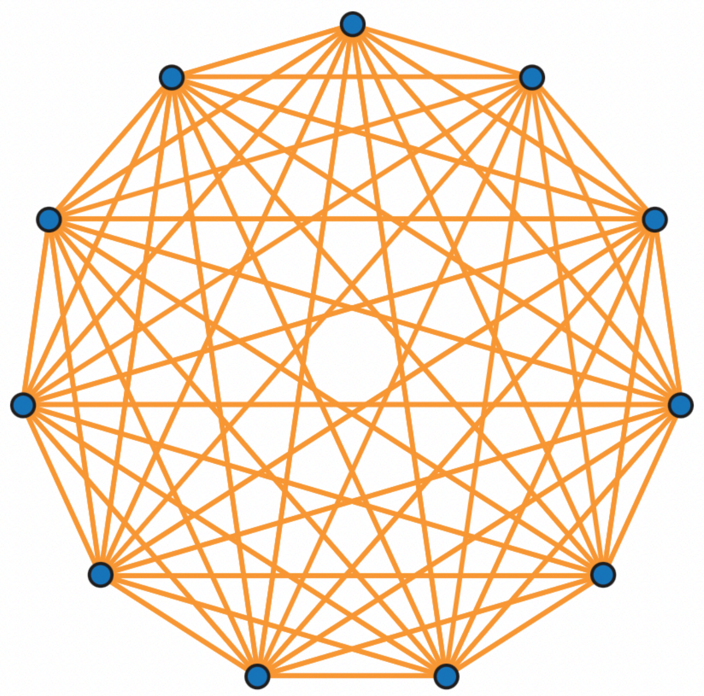

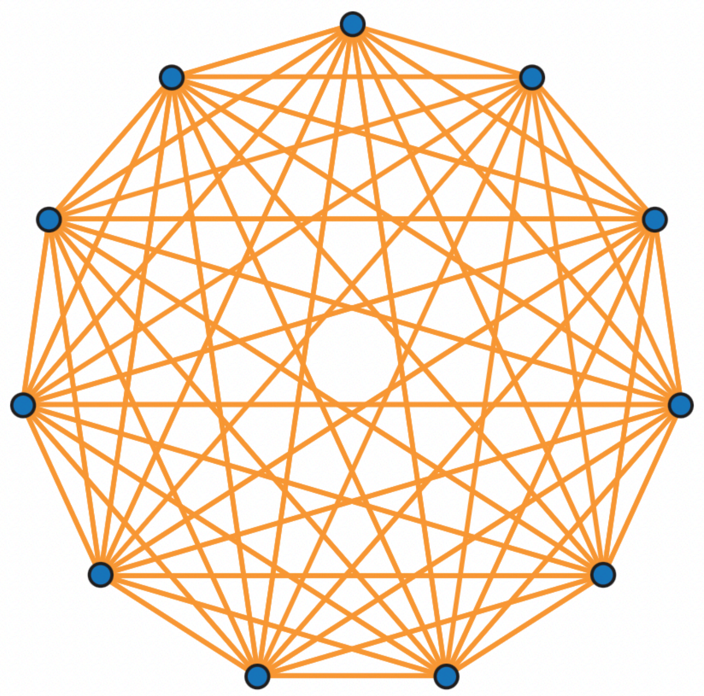

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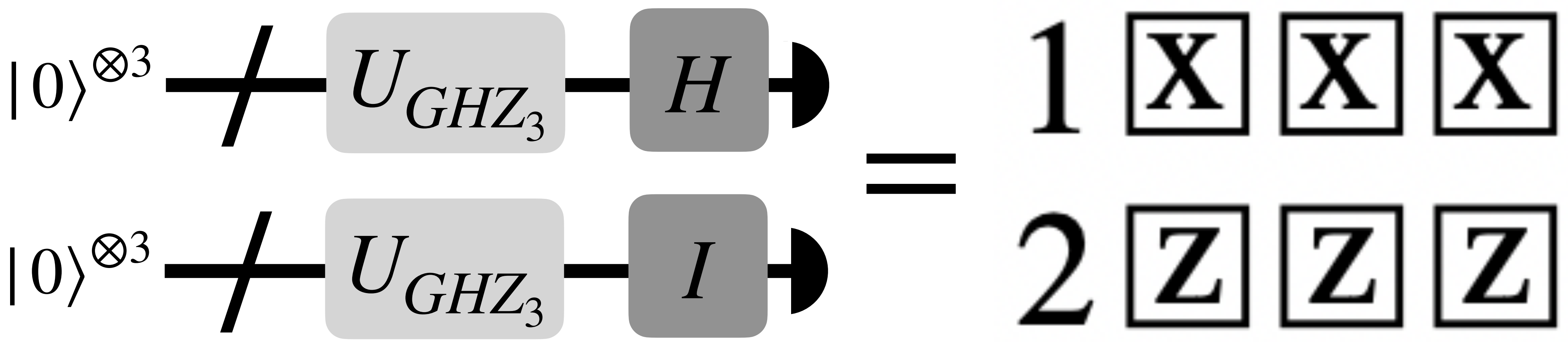
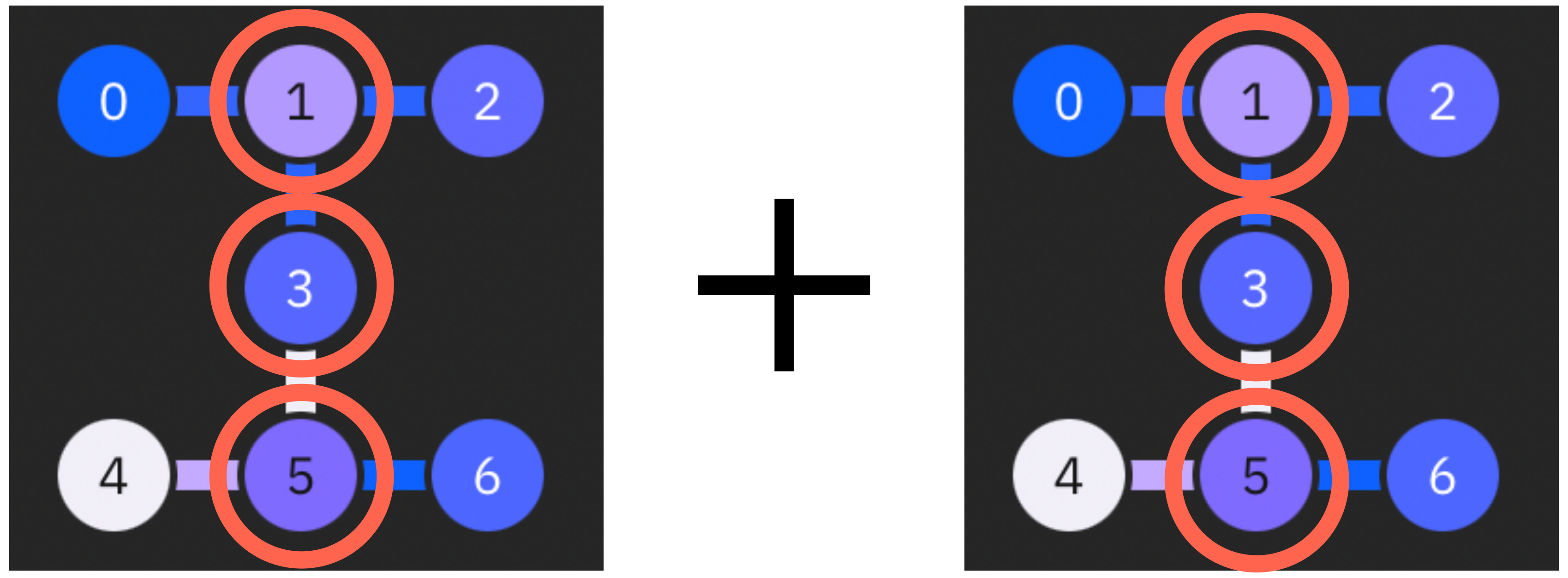



IBM Quantum ibm_nairobi

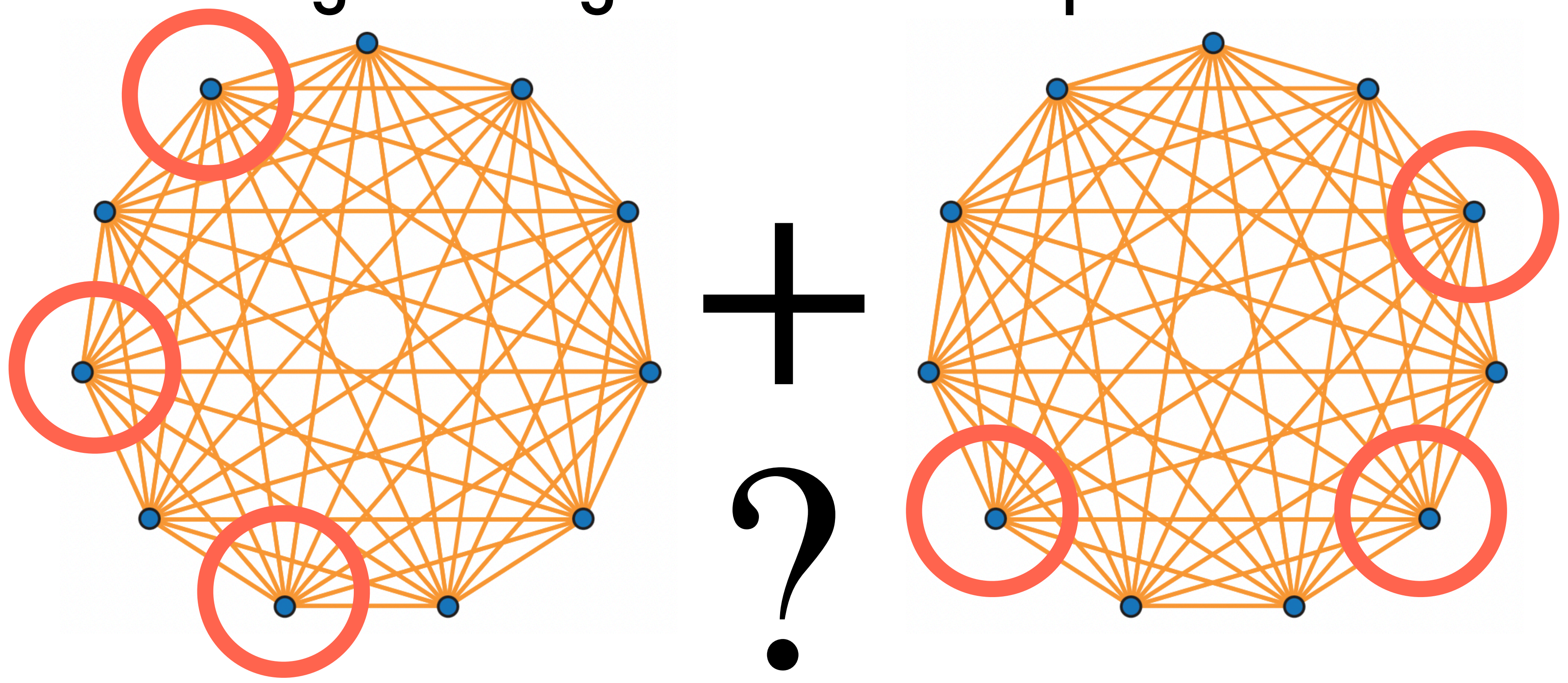
entanglement generation on qubits 1,3,5




$\mathrm{Tr}_{1,3,5}[\rho Z^{\otimes 3}]$ $\mathrm{Tr}_{1,3,5}[\rho X^{\otimes 3}]$

IonQ — Harmony

entanglement generation on qubits … ???




$\mathrm{Tr}_{0,2,4}[\rho Z^{\otimes 3}]$ $\mathrm{Tr}_{3,6,8}[\rho X^{\otimes 3}]$

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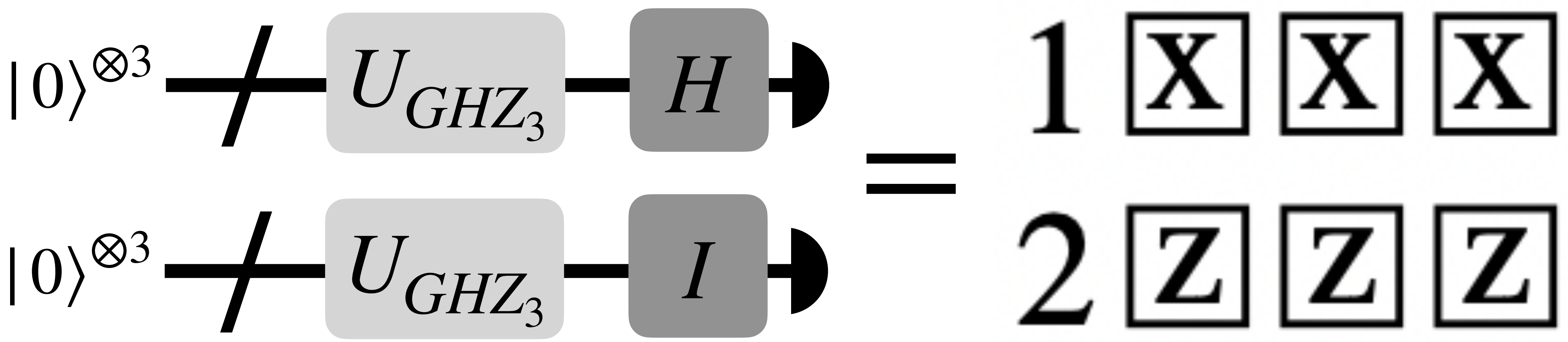


IBM Quantum ibm_nairobi

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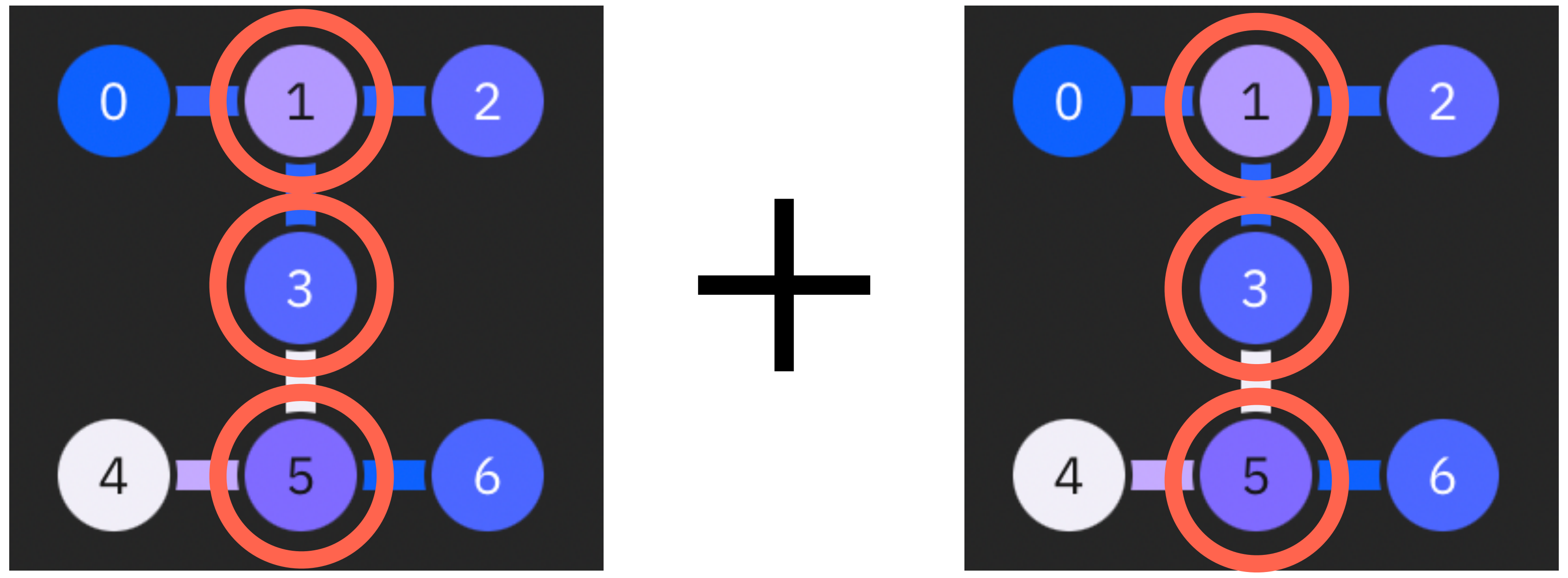


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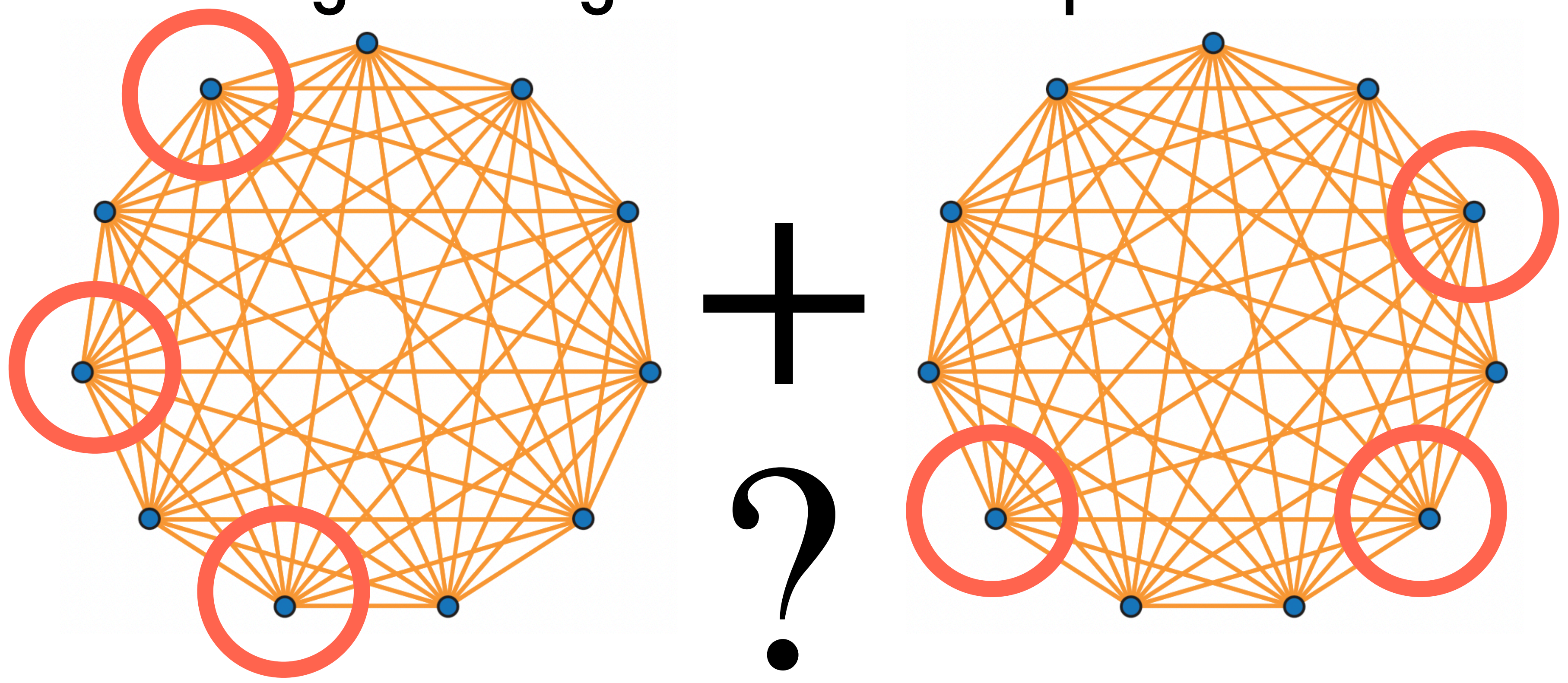


$\mathrm{Tr}_{0,2,4}[\rho Z^{\otimes 3}]$ $\mathrm{Tr}_{3,6,8}[\rho X^{\otimes 3}]$

**Without trust in qubit allocation, entanglement certification should be carried out at one shot**

# Main Idea: Entanglement Witnessing Circuit (EWC)

In cloud-based quantum computing services,

- Is it possible that the EW is measured in a single measurement ?

  Yes, with EWC Framework

- How to compose such a quantum circuit performing the one-shot measurement?

  Theoretical framework: Structural Physical Approximation (SPA)

  State Purification

- Does the size of such a circuit scale reasonably with # of qubits?

  Scale issue: # of fundamental gates required to compose a general #-qubit unitary

## Conventional Scheme

**Breakdown of an EW into Measurements**

$$W = \sum_i c_i M_i$$

Measurements realizable in laboratories

**Collection of measurement outcomes**

$$\rho \longrightarrow \boxed{\text{Dynamics}} \longrightarrow M_1 \qquad \mathrm{Tr}[\rho M_1]$$

$$\vdots$$

$$\rho \longrightarrow \boxed{\text{Dynamics}} \longrightarrow M_i \qquad \mathrm{Tr}[\rho M_i]$$

**Classical Post-Processing**

$$\mathrm{Tr}[\rho W] = \sum_i c_i \mathrm{Tr}[\rho M_i]$$

## Conventional Scheme

| Breakdown of an EW into Measurements | Collection of measurement outcomes | Classical Post-Processing |
| --- | --- | --- |
| $W = \sum_i c_i M_i$ | $\rho \to \boxed{\text{Dynamics}} \to M_1 \quad \mathrm{Tr}[\rho M_1]$ | $\mathrm{Tr}[\rho W] = \sum_i c_i \mathrm{Tr}[\rho M_i]$ |
| | $\vdots$ | |
| Measurements realizable in laboratories | $\rho \to \boxed{\text{Dynamics}} \to M_i \quad \mathrm{Tr}[\rho M_i]$ | # of measurement settings<br>Qubit Allocation |

## Conventional Scheme

| Breakdown of an EW into Measurements | Collection of measurement outcomes | Classical Post-Processing |
| --- | --- | --- |

$$W = \sum_i c_i M_i$$

Measurements realizable in laboratories

$$\rho \to \boxed{\text{Dynamics}} \to M_1 \quad \mathrm{Tr}[\rho\, M_1]$$

$$\vdots$$

$$\rho \to \boxed{\text{Dynamics}} \to M_i \quad \mathrm{Tr}[\rho\, M_i]$$

$$\mathrm{Tr}[\rho\, W] = \sum_i c_i \mathrm{Tr}[\rho\, M_i]$$

# of measurement settings

Qubit Allocation

---

## EWC Framework

1. Rendering of an EW into a positive operator via **Structural Physical Approximation** (SPA)

$$W \xrightarrow[\text{SPA}]{} \widetilde{W} : \text{quantum state}$$

(cf.) Entanglement Witness

J. Bae, Rep. Prog. Phys. 80 104001 (2017)

## Conventional Scheme

**Breakdown of an EW into Measurements**

$$W = \sum_i c_i M_i$$

Measurements realizable in laboratories

**Collection of measurement outcomes**

$$\rho \longrightarrow \boxed{\text{Dynamics}} \longrightarrow M_1 \qquad \mathrm{Tr}[\rho M_1]$$

$$\vdots$$

$$\rho \longrightarrow \boxed{\text{Dynamics}} \longrightarrow M_i \qquad \mathrm{Tr}[\rho M_i]$$

**Classical Post-Processing**

$$\mathrm{Tr}[\rho W] = \sum_i c_i \mathrm{Tr}[\rho M_i]$$

# of measurement settings

Qubit Allocation

---

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1. Rendering of an EW into a positive operator via **Structural Physical Approximation** (SPA)

$$W \xrightarrow[\text{SPA}]{} \widetilde{W} : \text{quantum state}$$

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2. **Purification** of the SPAed positive operator $\widetilde{W} \in \mathscr{H}_S \xrightarrow[\text{Purification}]{} |W\rangle \in \mathscr{H}_S \otimes \mathscr{H}_A$

J. Bae, Rep. Prog. Phys. 80 104001 (2017)

## Conventional Scheme

**Breakdown of an EW into Measurements**

$$W = \sum_i c_i M_i$$

Measurements realizable in laboratories

**Collection of measurement outcomes**

$$\rho \rightarrow \boxed{\text{Dynamics}} \rightarrow M_1 \qquad \mathrm{Tr}[\rho M_1]$$

$$\vdots$$

$$\rho \rightarrow \boxed{\text{Dynamics}} \rightarrow M_i \qquad \mathrm{Tr}[\rho M_i]$$

**Classical Post-Processing**

$$\mathrm{Tr}[\rho W] = \sum_i c_i \mathrm{Tr}[\rho M_i]$$

# of measurement settings

Qubit Allocation

---

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2. **Purification** of the SPAed positive operator $\widetilde{W} \in \mathscr{H}_S \xrightarrow[\text{Purification}]{} |W\rangle \in \mathscr{H}_S \otimes \mathscr{H}_A$

3. Design of **a quantum circuit** realizing the purified observable $|W\rangle$

$$|W\rangle = U_{|W\rangle}\left(|\mathbf{0}\rangle_S \otimes |\mathbf{0}\rangle_A\right)$$

J. Bae, Rep. Prog. Phys. 80 104001 (2017)

## Conventional Scheme

**BREAKDOWN OF AN EW INTO MEASUREMENTS**

$$W = \sum_i c_i M_i$$

Measurements realizable in laboratories

**COLLECTION OF MEASUREMENT OUTCOMES**

$\rho$ → Dynamics → $M_1$ $\quad \mathrm{Tr}[\rho\, M_1]$

⋮

$\rho$ → Dynamics → $M_i$ $\quad \mathrm{Tr}[\rho\, M_i]$

**CLASSICAL POST-PROCESSING**

$$\mathrm{Tr}[\rho\, W] = \sum_i c_i \mathrm{Tr}[\rho\, M_i]$$

# of measurement settings

Qubit Allocation

---

## EWC Framework

$$\mathrm{Tr}[\rho\, W] = \mathrm{Tr}_A \left[ U_W^\dagger (\cdots) U_W \left[ |\mathbf{0}\rangle\langle \mathbf{0}|_S \otimes I_A \right] \right]$$

$$|W\rangle = U_W \left( |\mathbf{0}\rangle_S \otimes |\mathbf{0}\rangle_A \right)$$

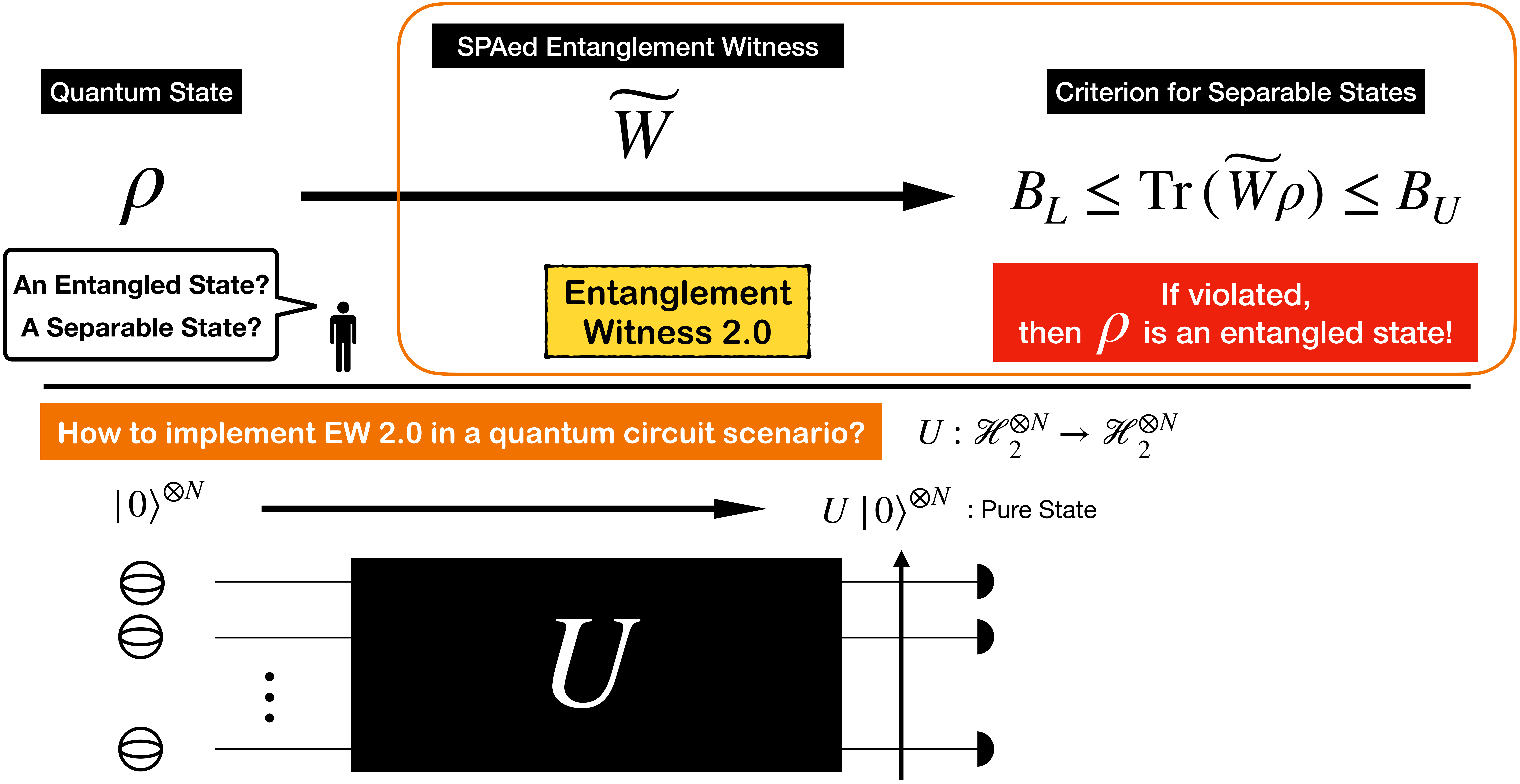
PREPARED STATE $\rho$

ANCILLAE $|\mathbf{0}\rangle_A$

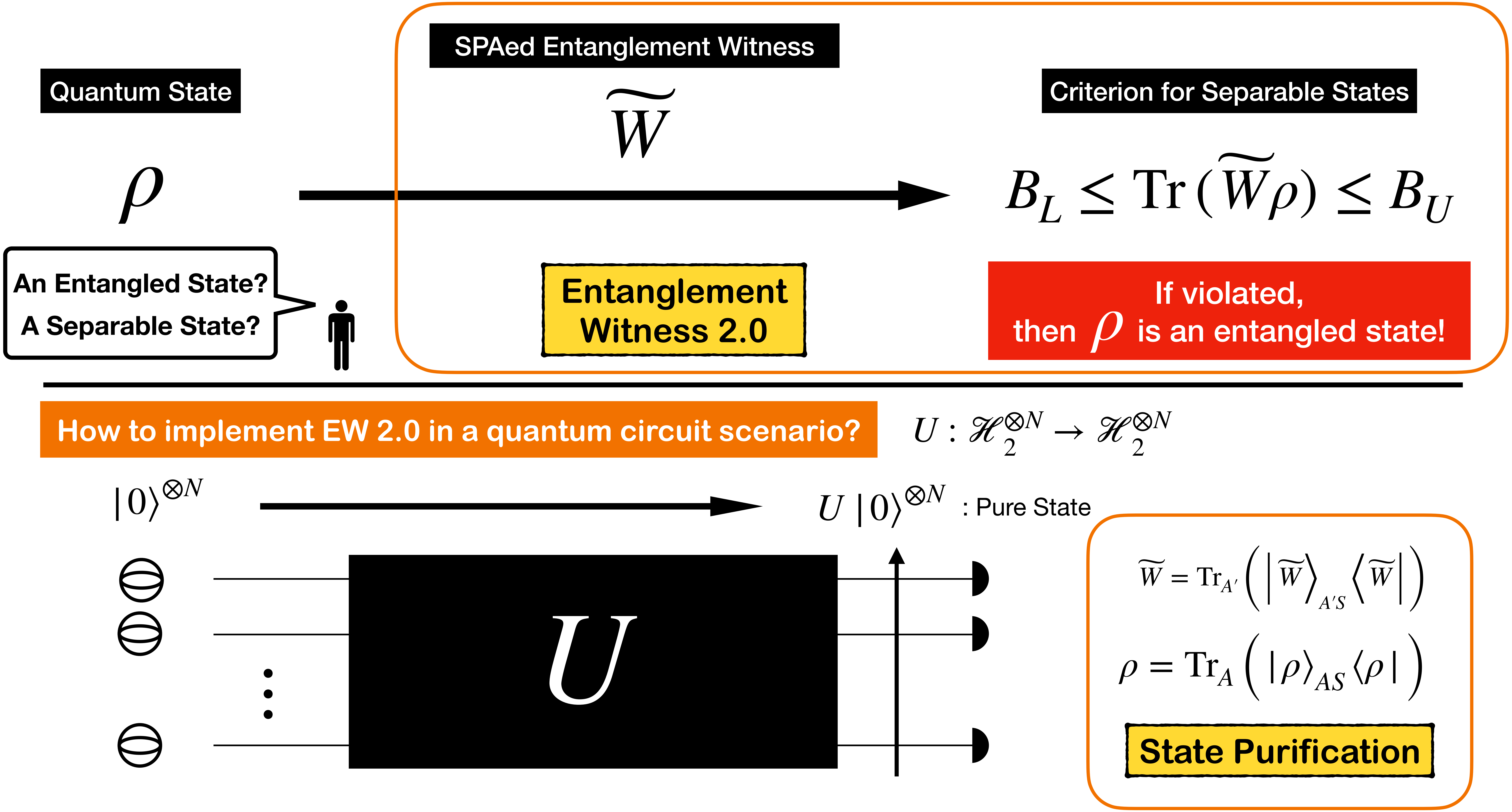
→ $U_W$ → $|\mathbf{0}\rangle\langle \mathbf{0}|_S \otimes I_A$

QUANTUM CIRCUIT OF PURIFIED EW

- EW Estimation in **One Attempt!**
- Lesser Quantum Resources in Measurements
- Immune to Qubit Allocation Issues in Quantum Computing Services



Quantum State
$\rho$
An Entangled State?
A Separable State?
SPAed Entanglement Witness
$\widetilde{W}$
Entanglement Witness 2.0
Criterion for Separable States
$B_L \leq \mathrm{Tr}(\widetilde{W}\rho) \leq B_U$
If violated,
then $\rho$ is an entangled state!
How to implement EW 2.0 in a quantum circuit scenario?
$U : \mathscr{H}_2^{\otimes N} \to \mathscr{H}_2^{\otimes N}$
$|0\rangle^{\otimes N}$
$U\,|0\rangle^{\otimes N}$ : Pure State
$U$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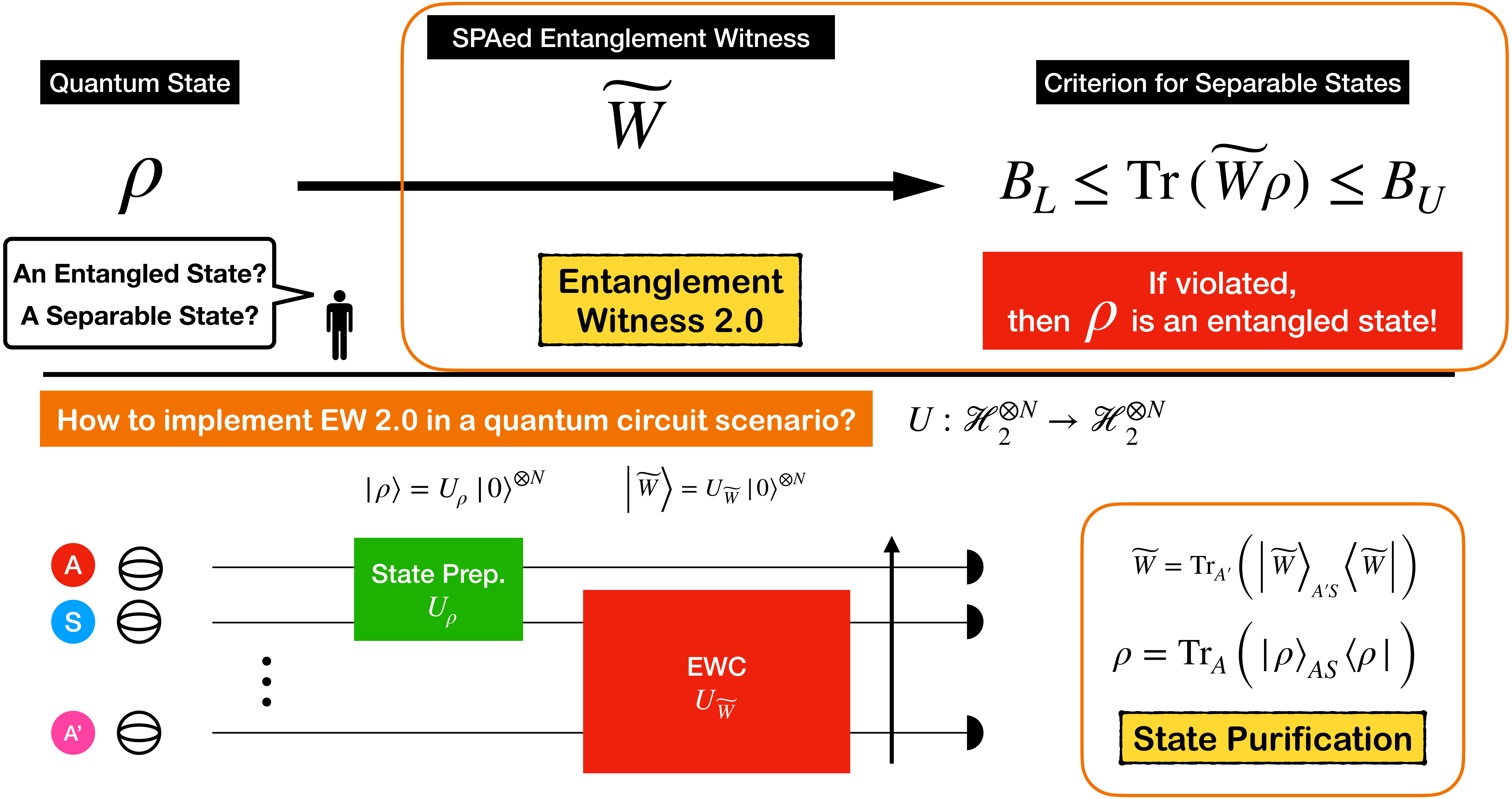
: Pure State
$U$
$\widetilde{W} = \mathrm{Tr}_{A'}\left(|\widetilde{W}\rangle_{A'S}\langle\widetilde{W}|\right)$
$\rho = \mathrm{Tr}_A\left(|\rho\rangle_{AS}\langle\rho|\right)$
State Purification



Quantum State
$\rho$
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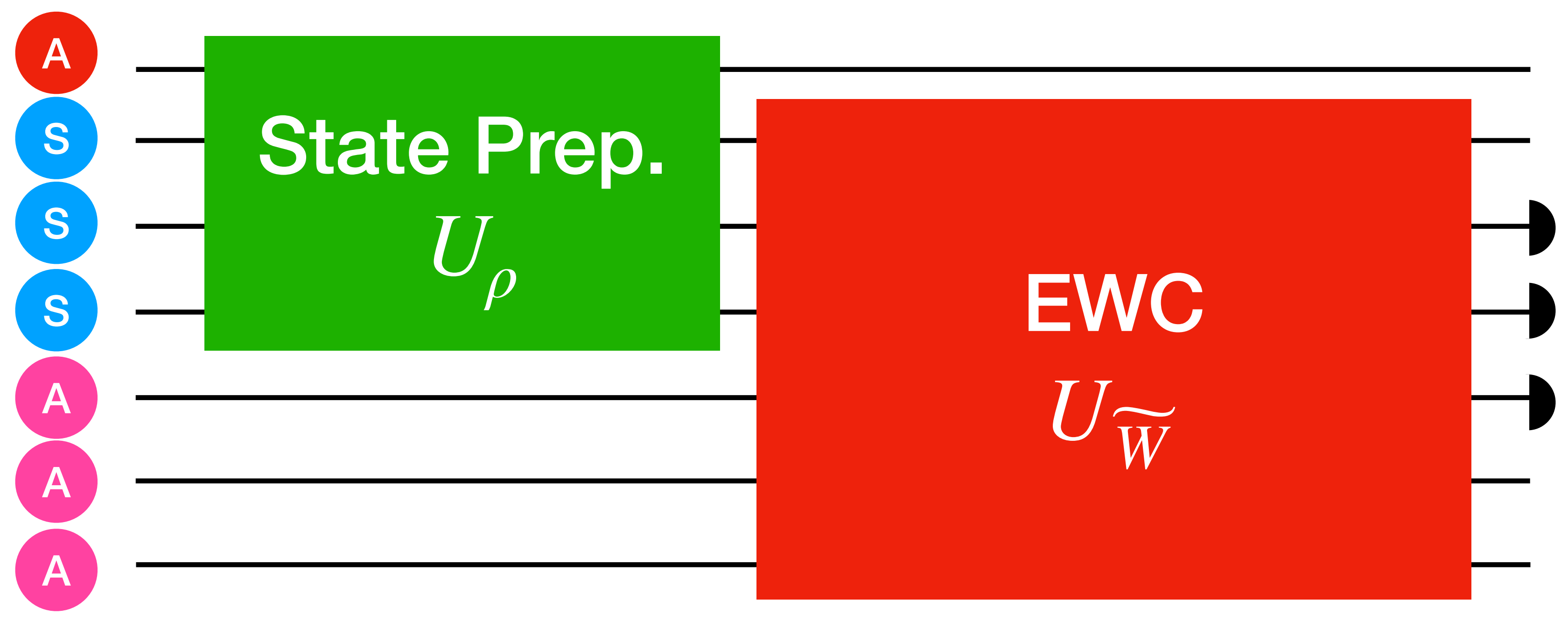
$\widetilde{W}$
Entanglement Witness 2.0
Criterion for Separable States
$B_L \leq \mathrm{Tr}\,(\widetilde{W}\rho) \leq B_U$
If violated, then $\rho$ is an entangled state!
How to implement EW 2.0 in a quantum circuit scenario?
$U : \mathscr{H}_2^{\otimes N} \rightarrow \mathscr{H}_2^{\otimes N}$
$|\rho\rangle = U_\rho\,|0\rangle^{\otimes N}$
$|\widetilde{W}\rangle = U_{\widetilde{W}}\,|0\rangle^{\otimes N}$
A
S
A'
State Prep. $U_\rho$
EWC $U_{\widetilde{W}}$
$\widetilde{W} = \mathrm{Tr}_{A'}\left(|\widetilde{W}\rangle_{A'S}\langle\widetilde{W}|\right)$
$\rho = \mathrm{Tr}_A\left(|\rho\rangle_{AS}\langle\rho|\right)$
State Purification

# Two Schemes for arranging parts of quantum circuits

# of Qubits in State Preparation = 3

## Scheme 1

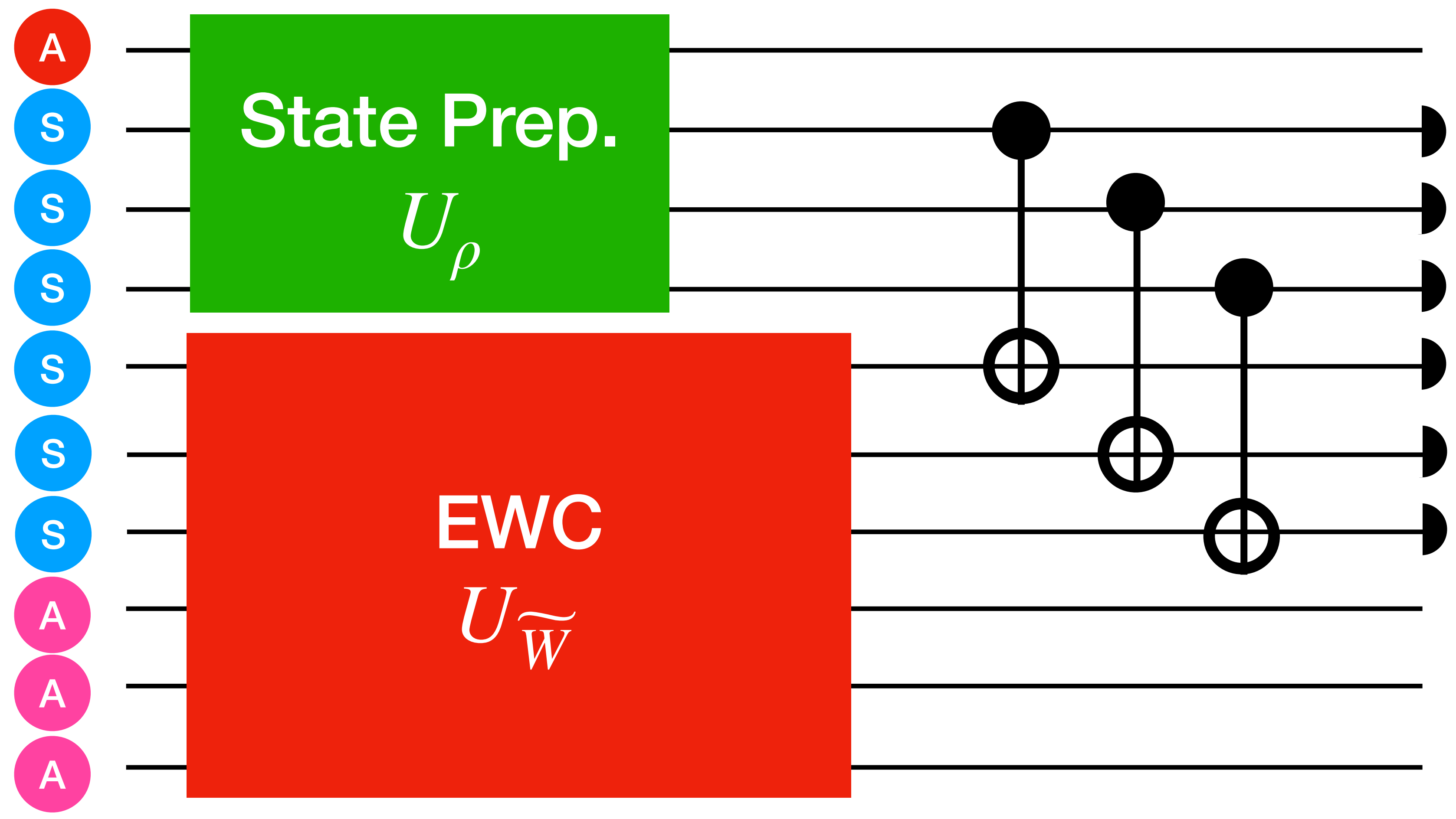



# of System Qubits = 3

# of Ancillae for State Preparation = 0~3

# of Ancillae for EWC = 3

## Scheme 2




# of System Qubits = 3 x 2 = 6

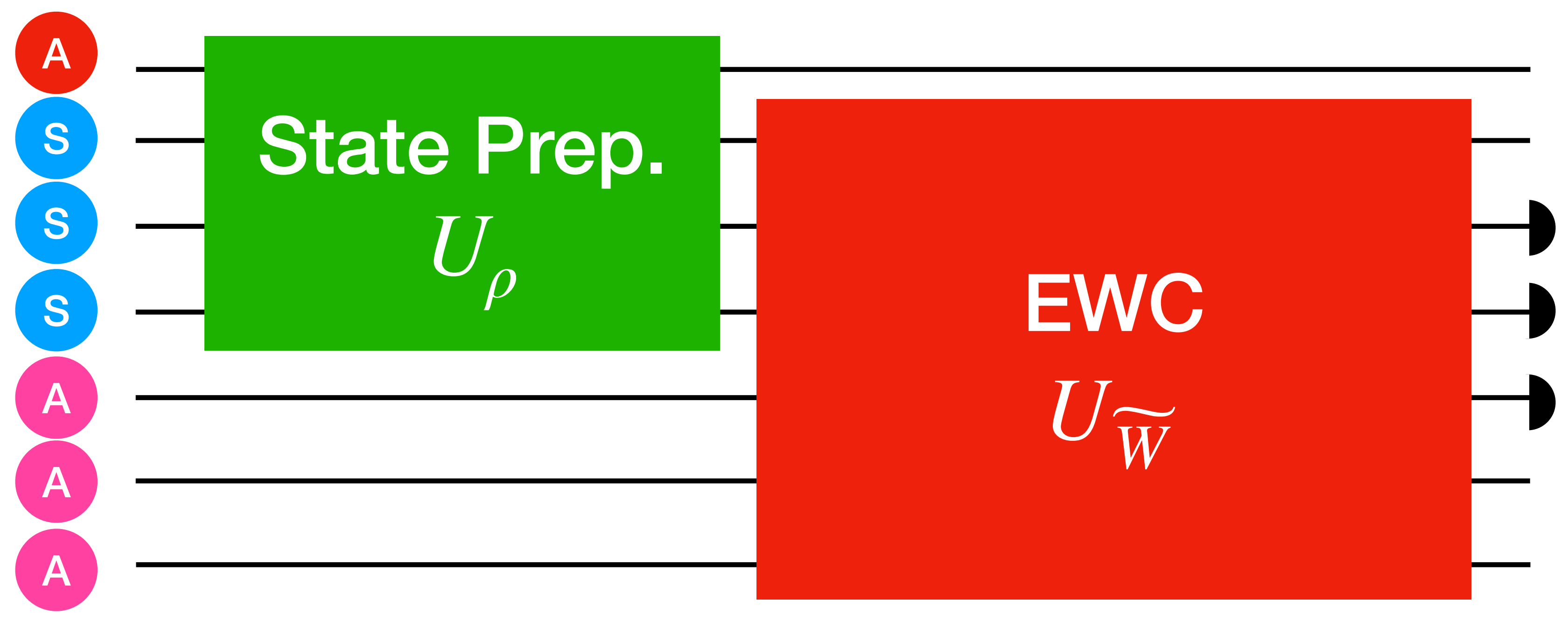
# of Ancillae for State Preparation = 0~3

# of Ancillae for EWC = 3

# Two Schemes for arranging parts of quantum circuits

# of Qubits in State Preparation = 3

## Scheme 1



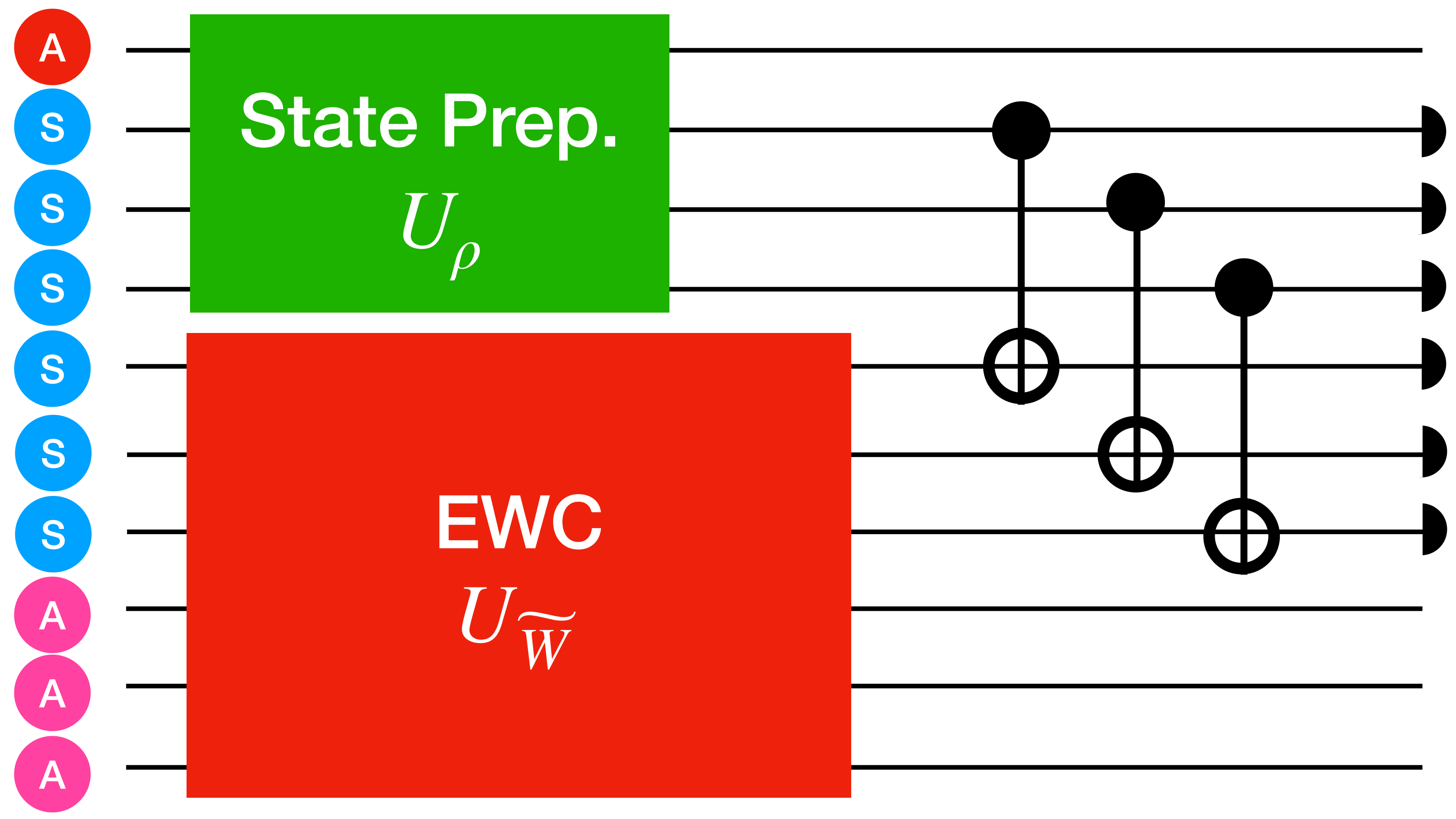

# of System Qubits = 3

# of Ancillae for State Preparation = 0~3

# of Ancillae for EWC = 3

# of Total Qubits is less needed

## Scheme 2




# of System Qubits = 3 x 2 = 6

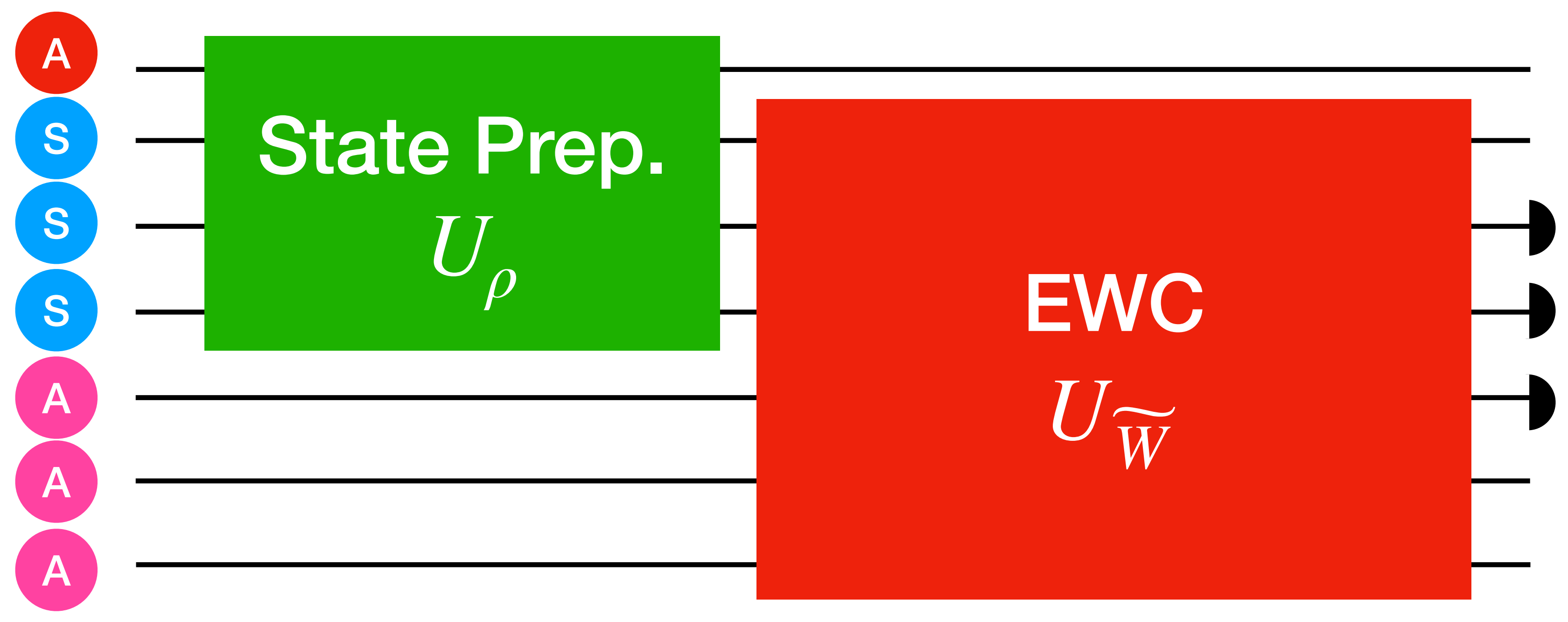
# of Ancillae for State Preparation = 0~3

# of Ancillae for EWC = 3

# Two Schemes for arranging parts of quantum circuits

**# of Qubits in State Preparation = 3**

## Scheme 1




**# of System Qubits = 3**

**# of Ancillae for State Preparation = 0~3**

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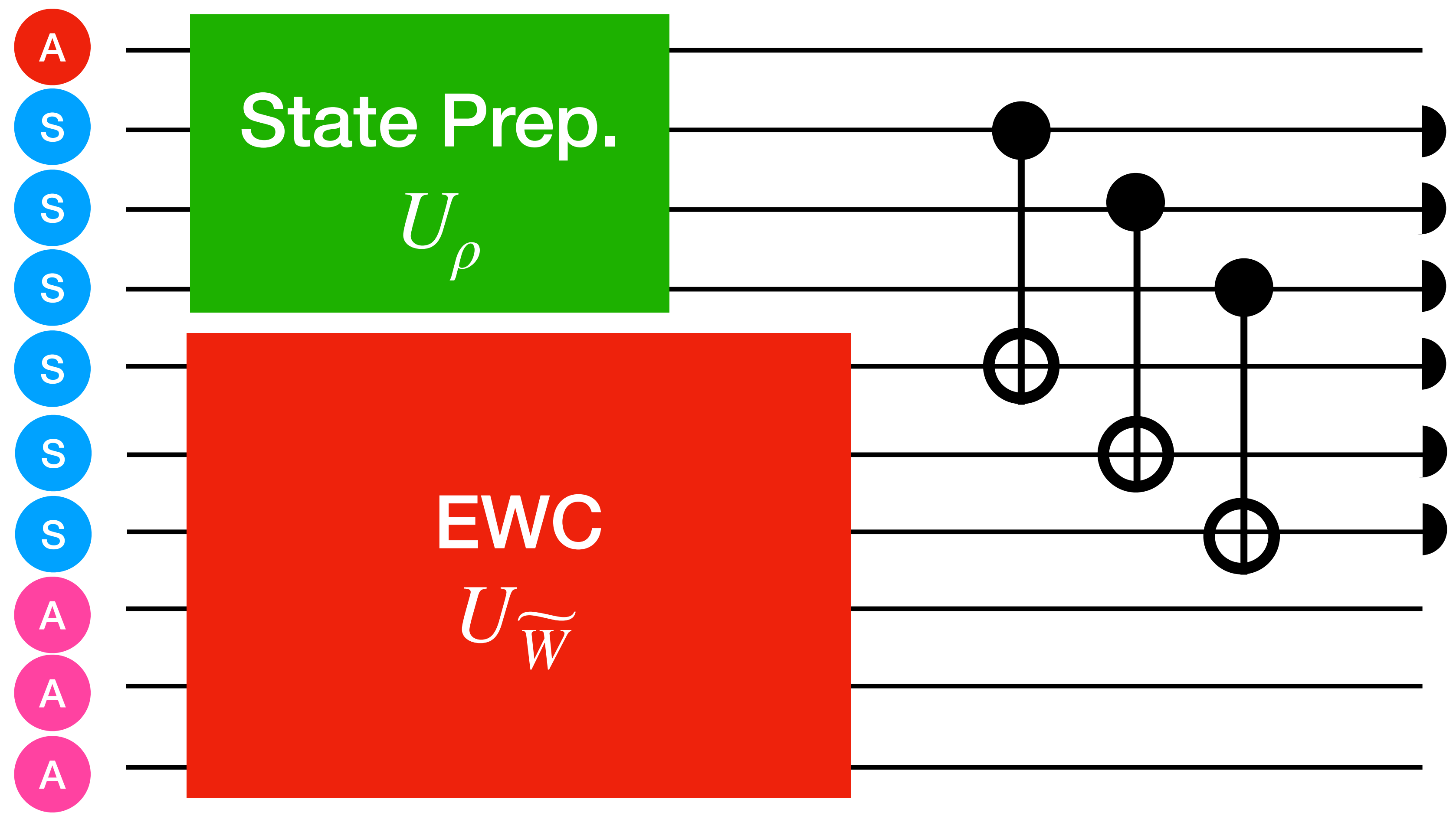
**# of Total Qubits is less needed**

## Scheme 2




**# of System Qubits = 3 x 2 = 6**

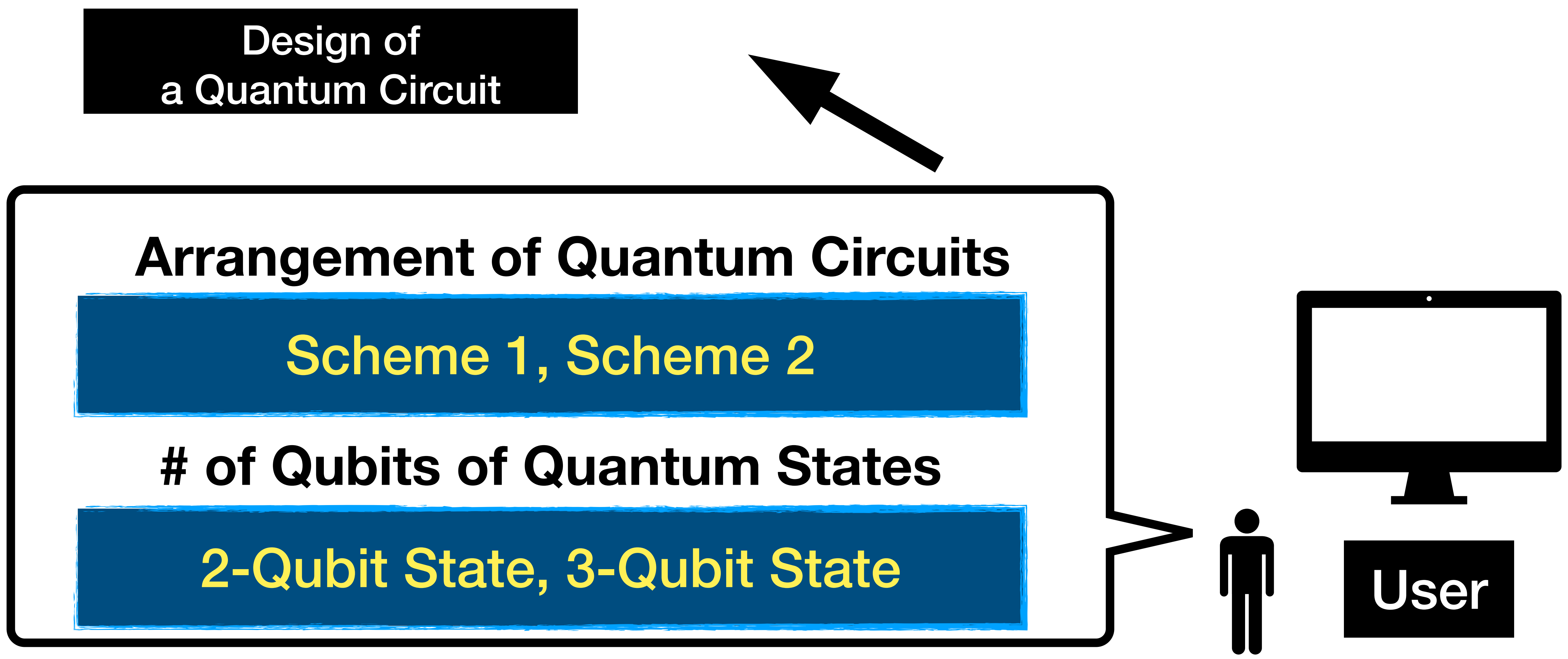
**# of Ancillae for State Preparation = 0~3**

**# of Ancillae for EWC = 3**

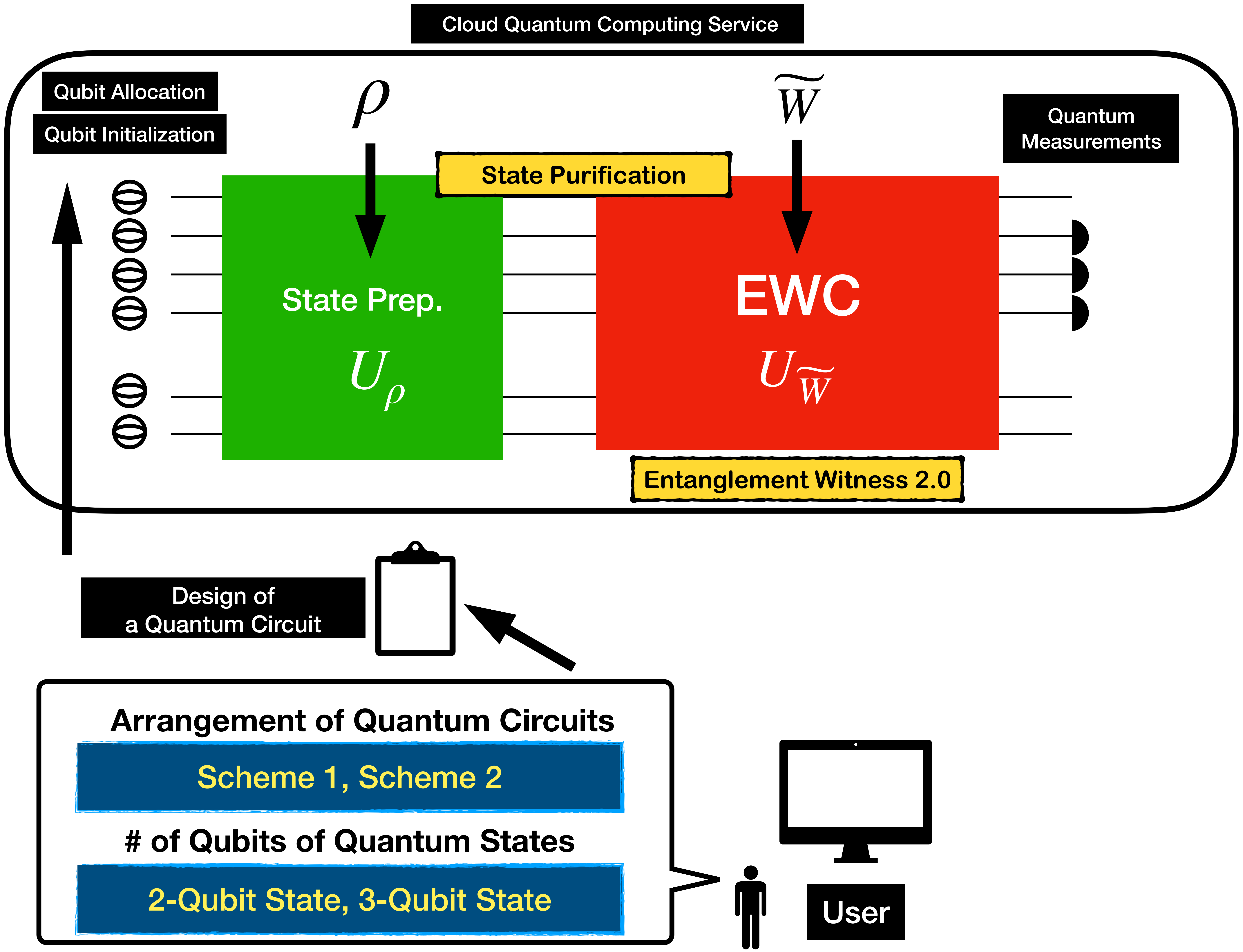
**Exploits the Idea of Quantum Teleportation**

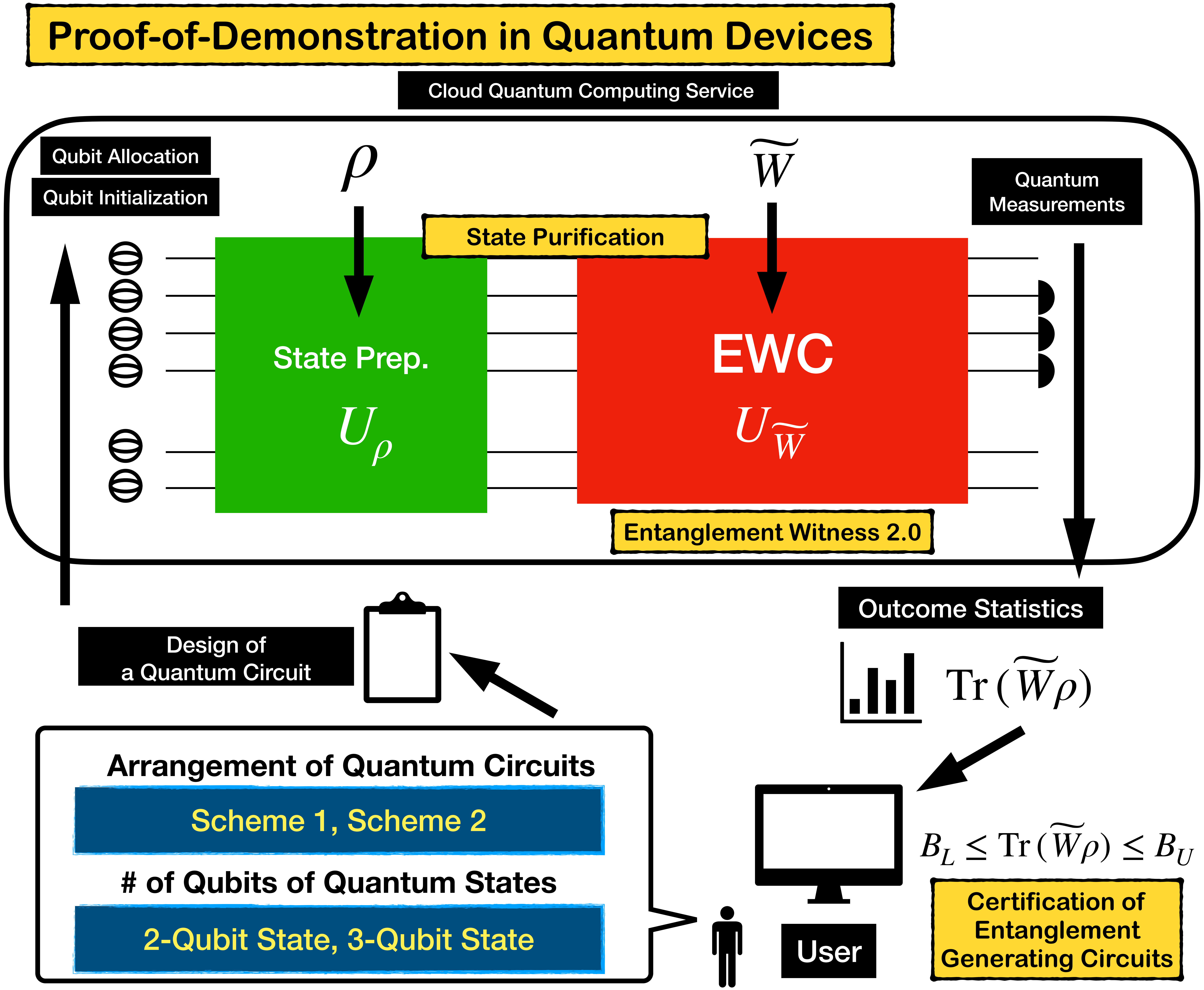
**The Circuit Depth can be Reduced**

# Proof-of-Demonstration in Quantum Devices



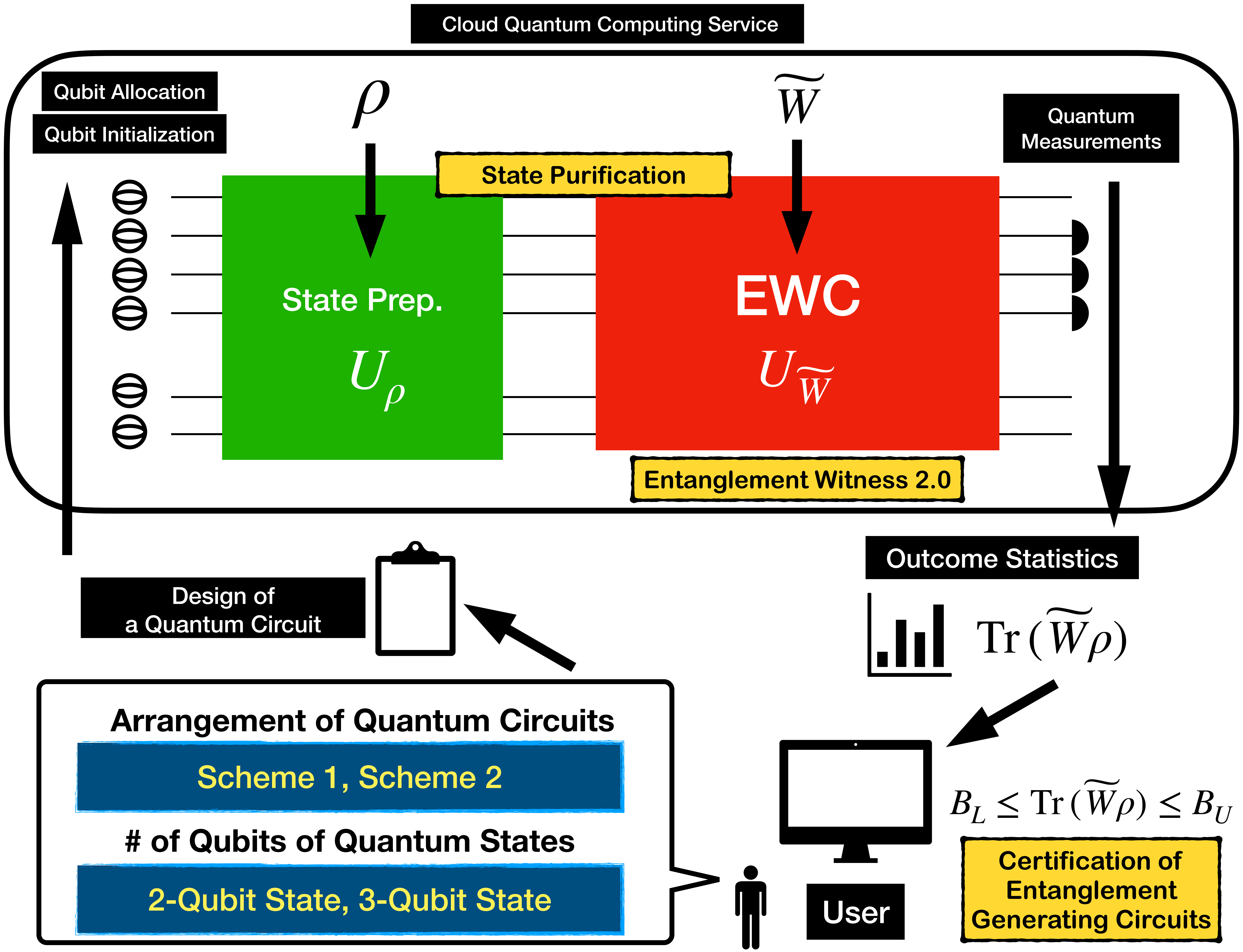
# Proof-of-Demonstration in Quantum Devices



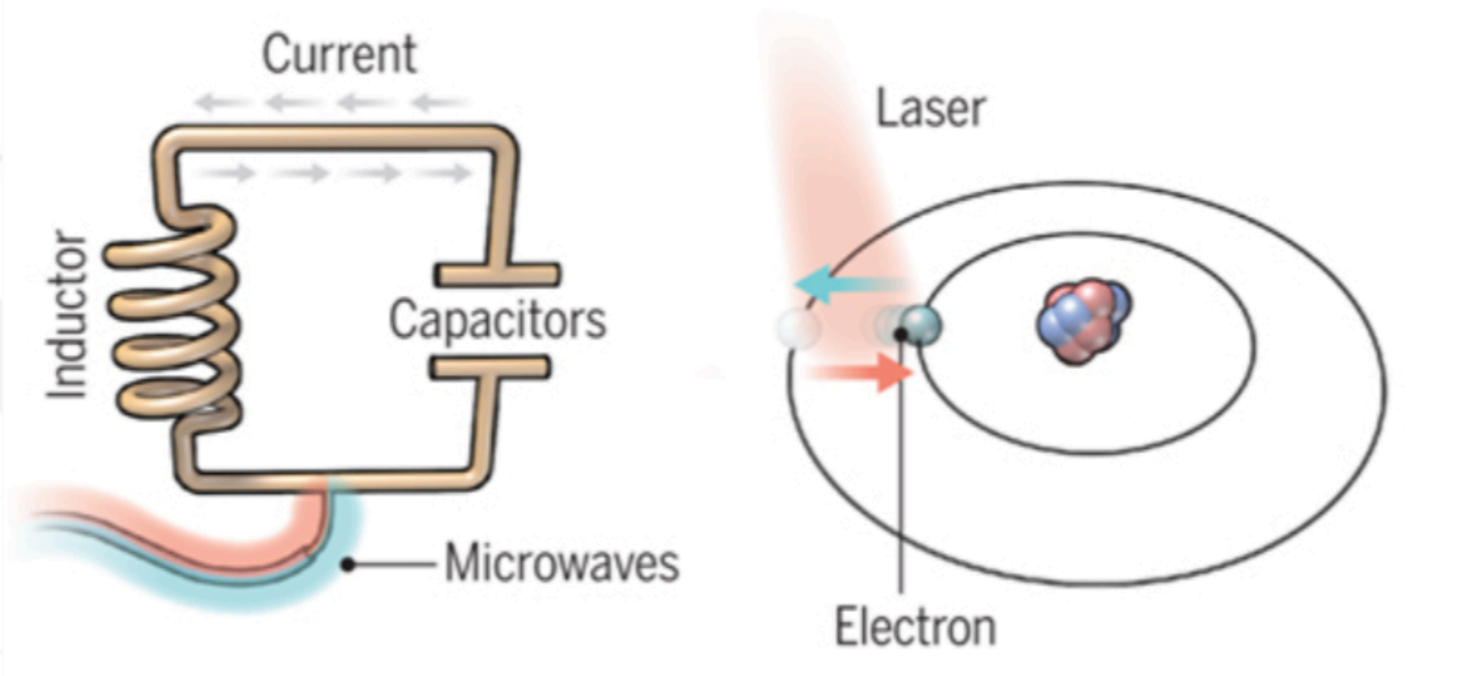


Proof-of-Demonstration in Quantum Devices
Cloud Quantum Computing Service
Qubit Allocation
Qubit Initialization
$\rho$
$\widetilde{W}$
Quantum Measurements
State Purification
State Prep.
$U_\rho$
EWC
$U_{\widetilde{W}}$
Entanglement Witness 2.0
Outcome Statistics
Design of a Quantum Circuit
$\mathrm{Tr}\,(\widetilde{W}\rho)$
Arrangement of Quantum Circuits
Scheme 1, Scheme 2
# of Qubits of Quantum States
2-Qubit State, 3-Qubit State
$B_L \leq \mathrm{Tr}\,(\widetilde{W}\rho) \leq B_U$
User
Certification of Entanglement Generating Circuits

# Proof-of-Demonstration in Quantum Devices

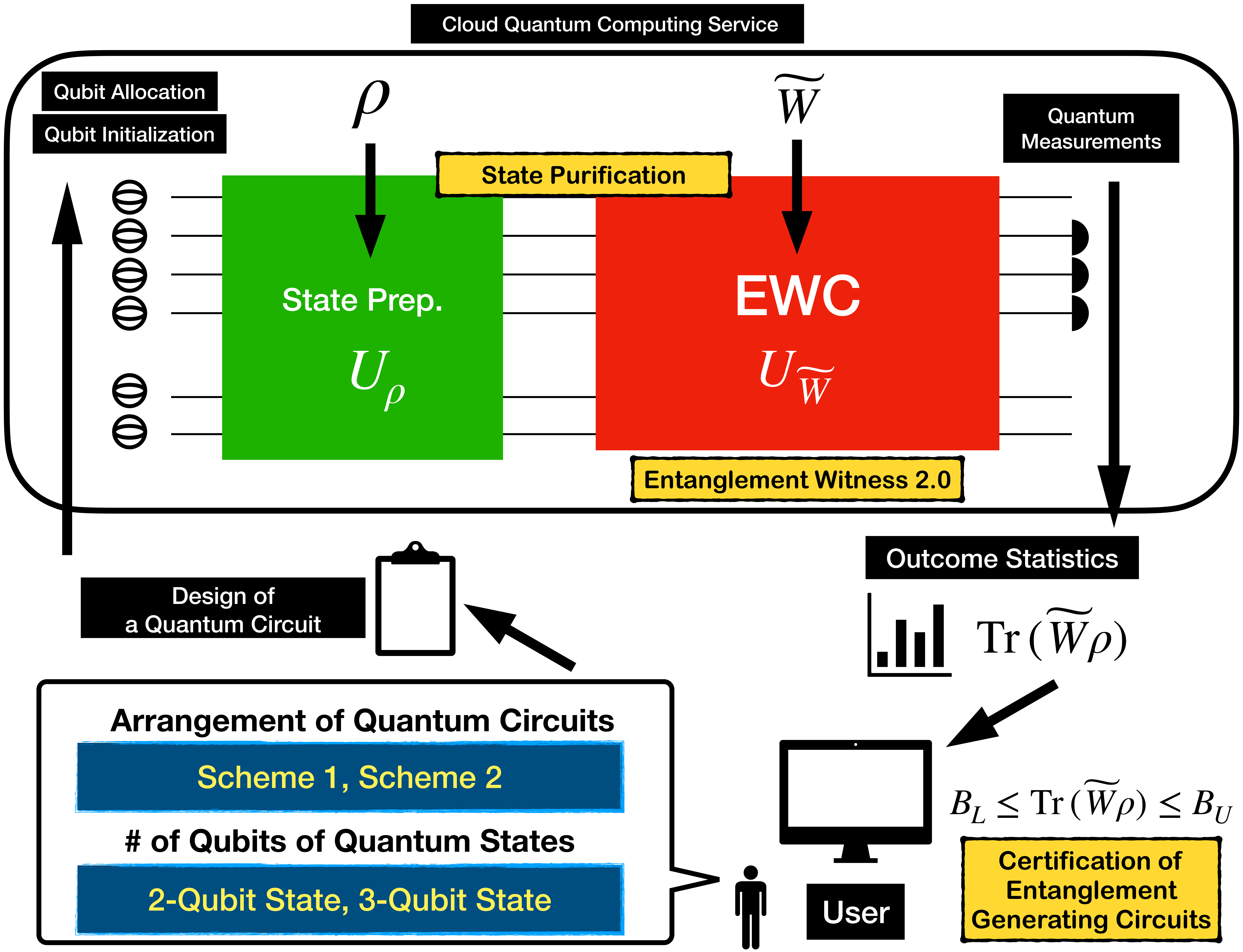



| | IBM Q | IonQ |
|---|---|---|
| | **Superconducting loops** | **Trapped ions** |
| | A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states. | Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states. |
| **Longevity** (seconds) | 0.00005 | >1000 |
| **Logic success rate** | 99.4% | 99.9% |
| **Number entangled** | 9 | 14 |
| **Company support** | Google, IBM, Quantum Circuits | ionQ |
| **Pros** | Fast working. Build on existing semiconductor industry. | Very stable. Highest achieved gate fidelities. |
| **Cons** | Collapse easily and must be kept cold. | Slow operation. Many lasers are needed. |

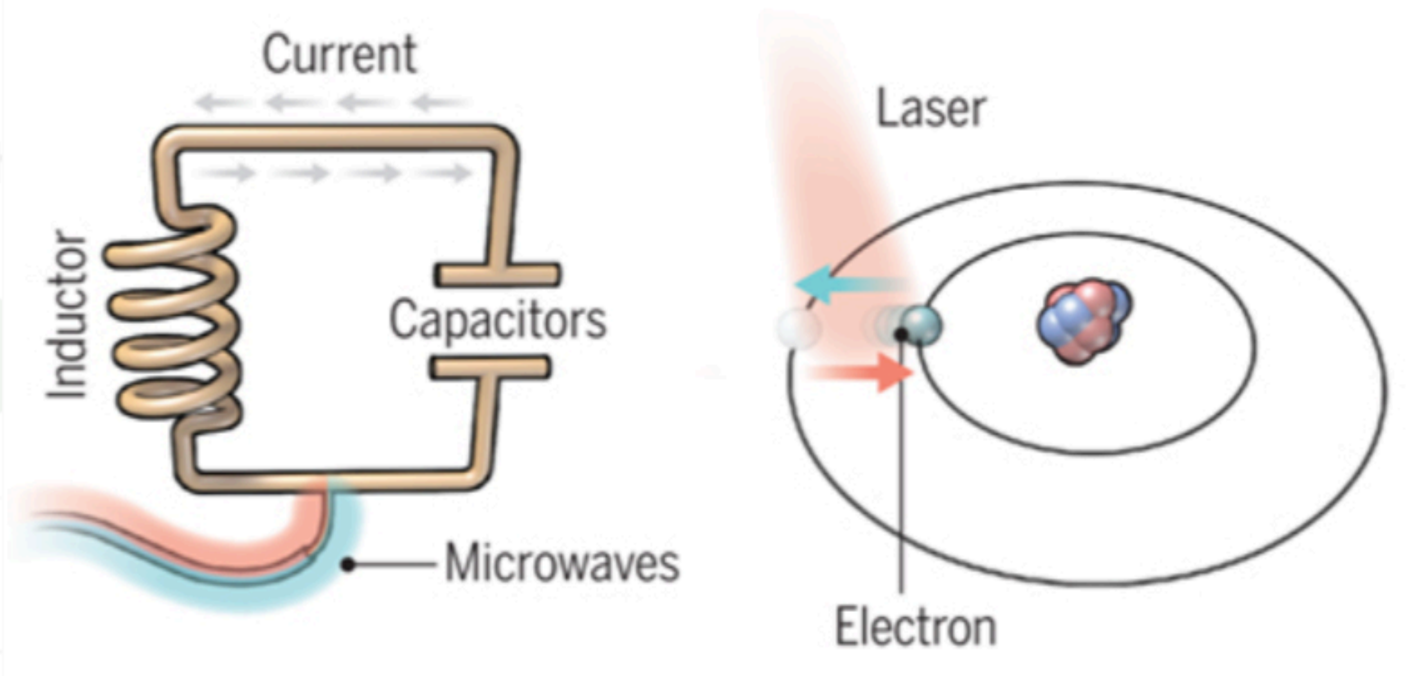



doi: 10.1126/science.aal0442

# Proof-of-Demonstration in Quantum Devices




IBM Q    IONQ




| Superconducting loops | Trapped ions |
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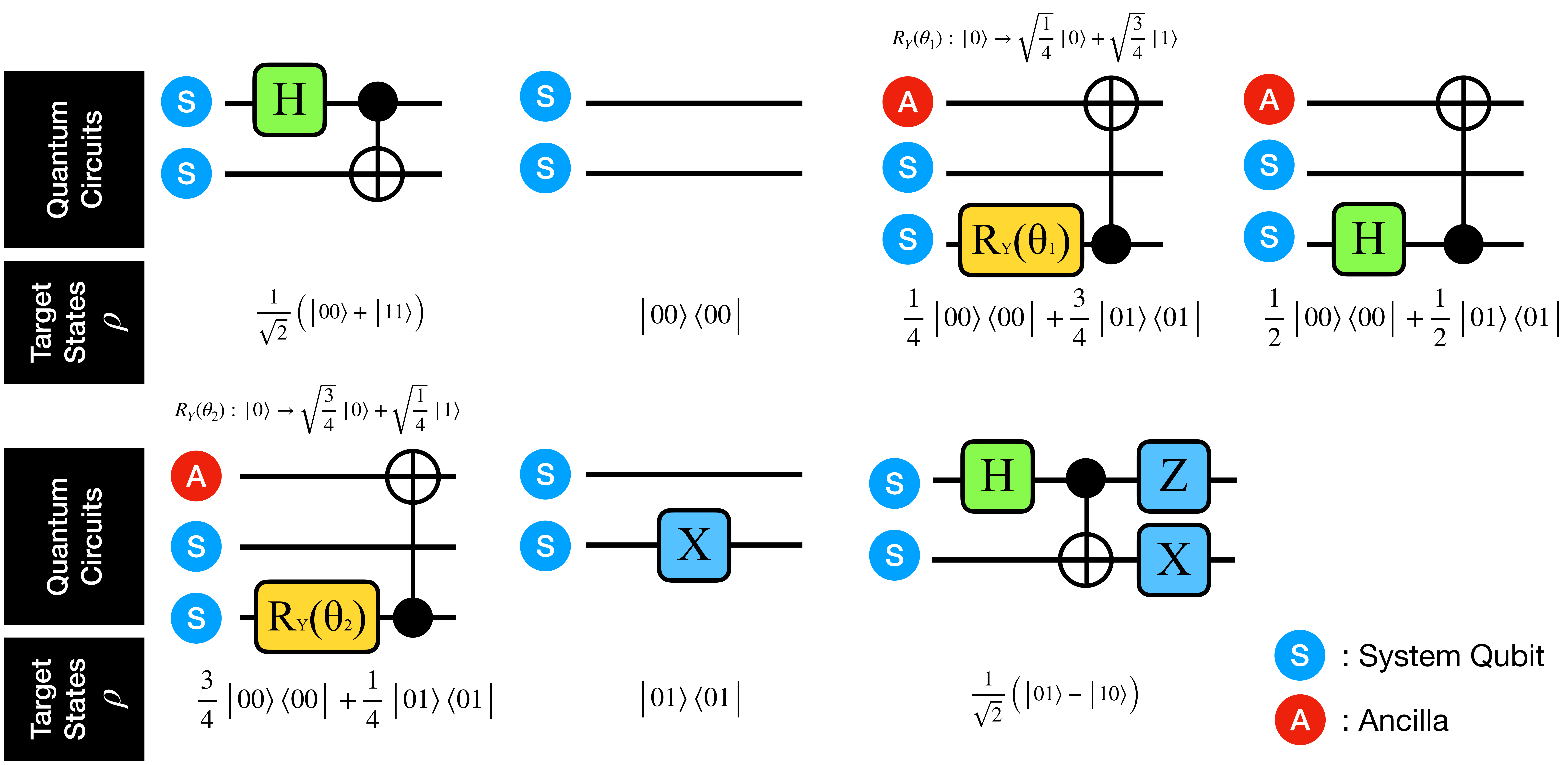
| **Pros** | |
| Fast working. Build on existing semiconductor industry. | Very stable. Highest achieved gate fidelities. |
| **Cons** | |
| Collapse easily and must be kept cold. | No Qubit Allocation Functionality ! |

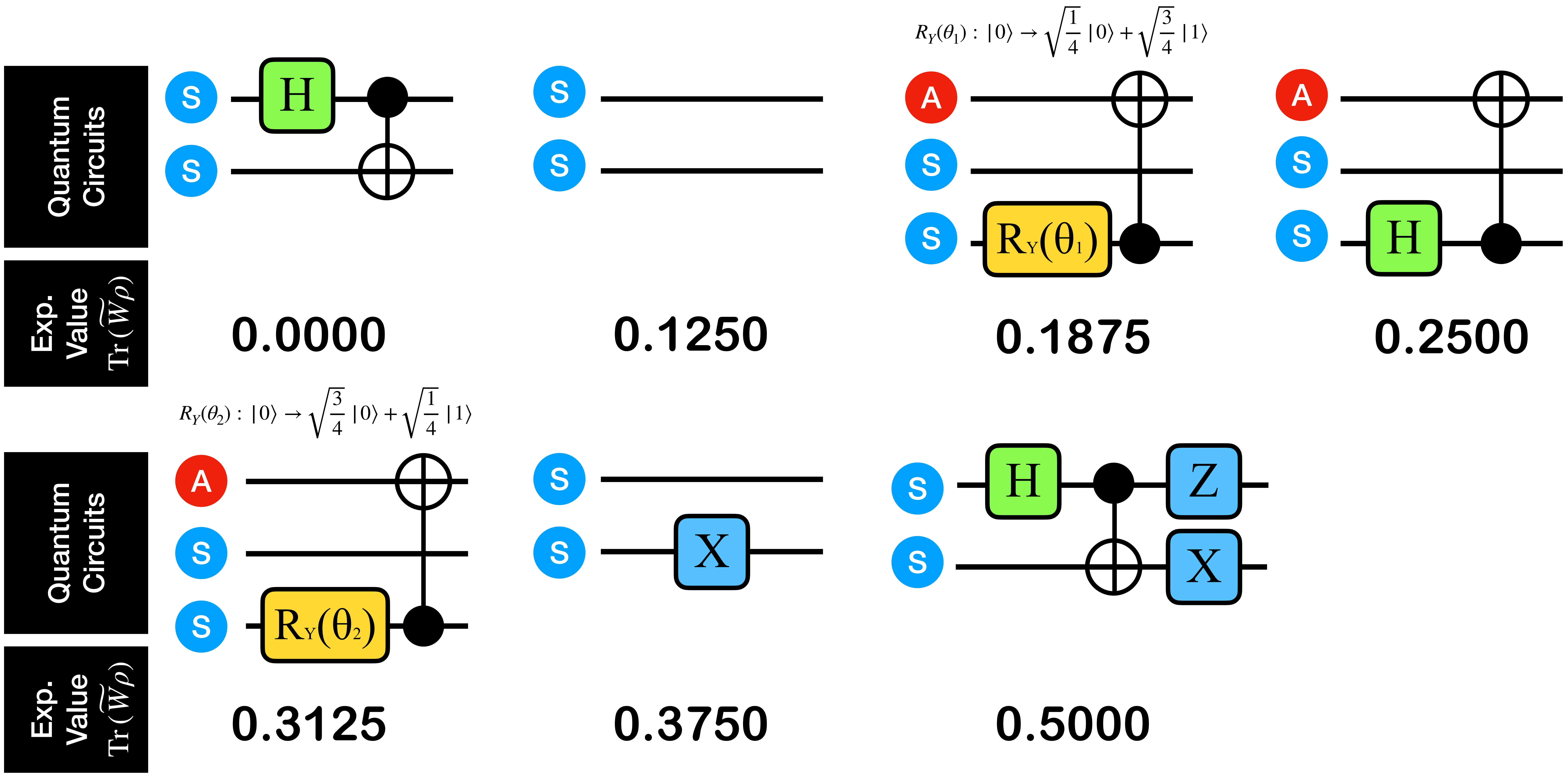
doi: 10.1126/science.aal0442

# Quantum Circuits with 2 qubits for demonstration



# Quantum Circuits with 2 qubits for demonstration

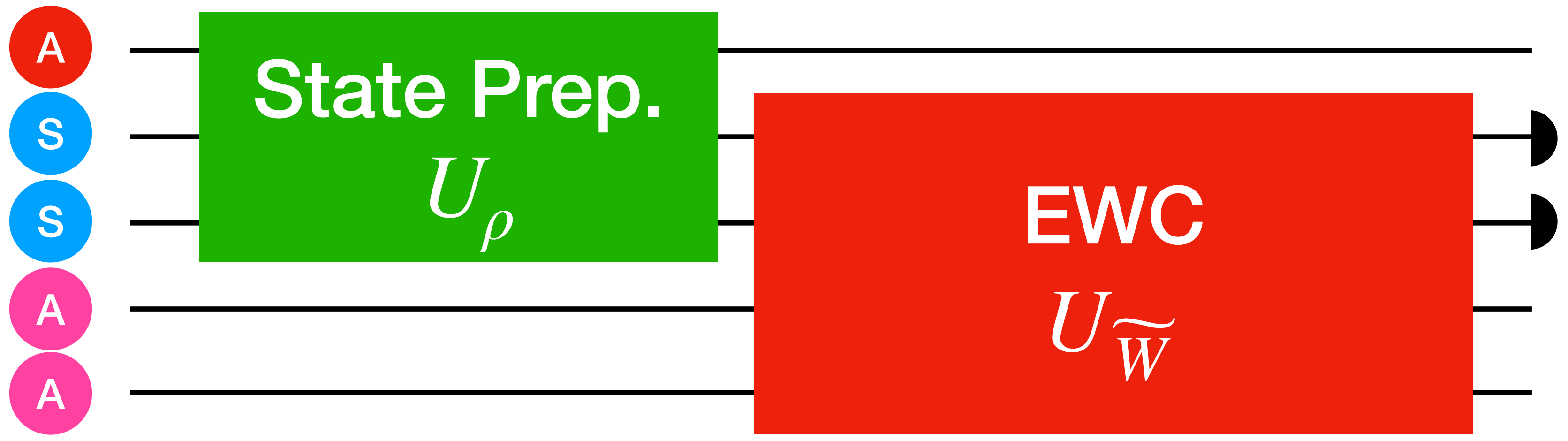
$$\widetilde{W} = \frac{1}{4}I - \frac{1}{4}\left(|\phi^+\rangle\langle\phi^+| - |\psi^-\rangle\langle\psi^-|\right)$$



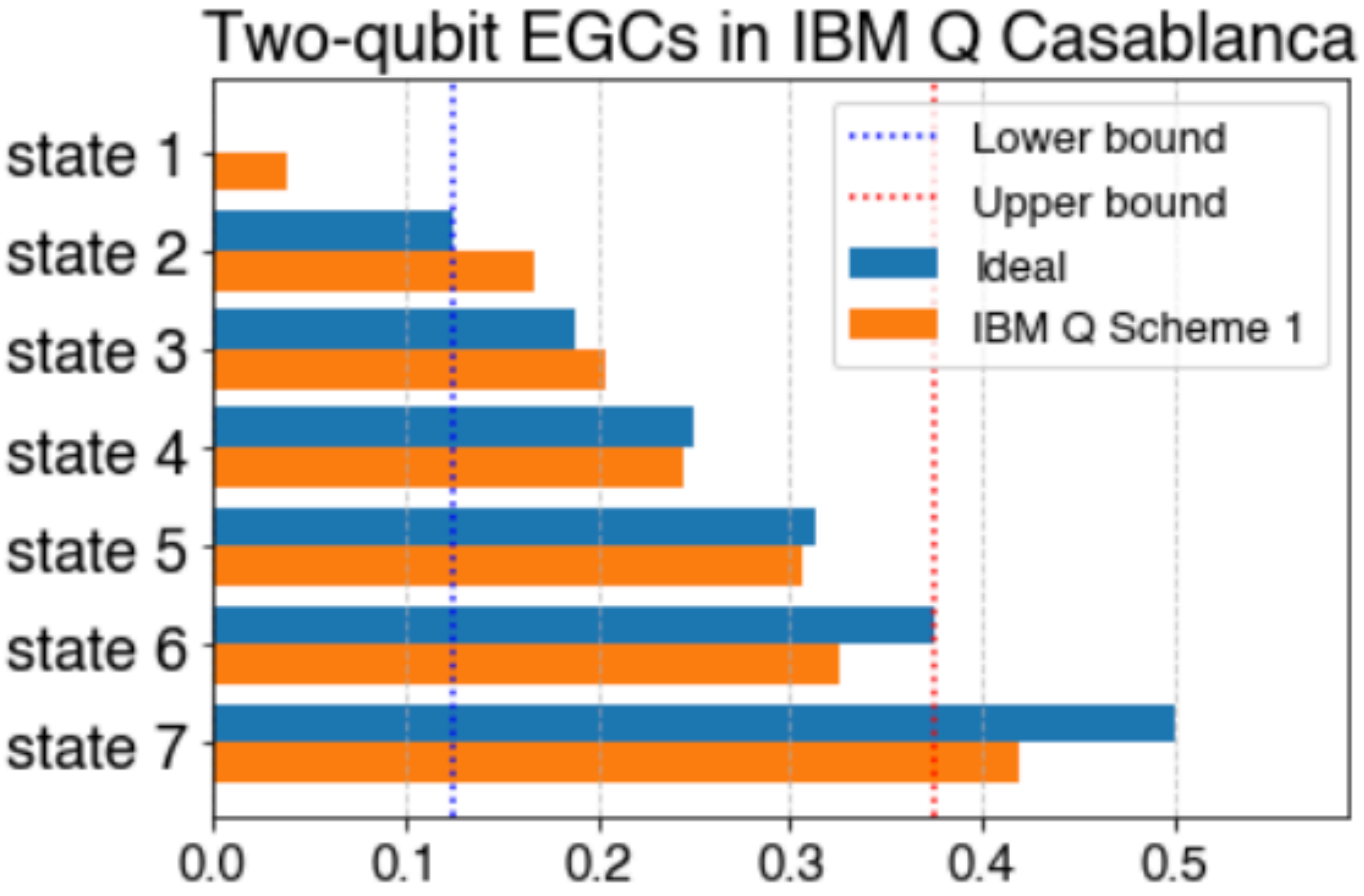

$$\widetilde{W} = \frac{1}{4}I - \frac{1}{4}\left(|\phi^+\rangle\langle\phi^+| - |\psi^-\rangle\langle\psi^-|\right)$$


| | | |
|---|---|---|
| **State 1** | $\frac{1}{\sqrt{2}}\left(\|00\rangle + \|11\rangle\right)$ | **0.0000** |
| **State 2** | $\|00\rangle\langle 00\|$ | **0.1250** |
| **State 3** | $\frac{1}{4}\|00\rangle\langle 00\| + \frac{3}{4}\|01\rangle\langle 01\|$ | **0.1875** |
| **State 4** | $\frac{1}{2}\|00\rangle\langle 00\| + \frac{1}{2}\|01\rangle\langle 01\|$ | **0.2500** |
| **State 5** | $\frac{3}{4}\|00\rangle\langle 00\| + \frac{1}{4}\|01\rangle\langle 01\|$ | **0.3125** |
| **State 6** | $\|01\rangle\langle 01\|$ | **0.3750** |
| **State 7** | $\frac{1}{\sqrt{2}}\left(\|01\rangle - \|10\rangle\right)$ | **0.5000** |

## 2-qubit demonstration in IBM Q Casablanca




**Scheme 1**

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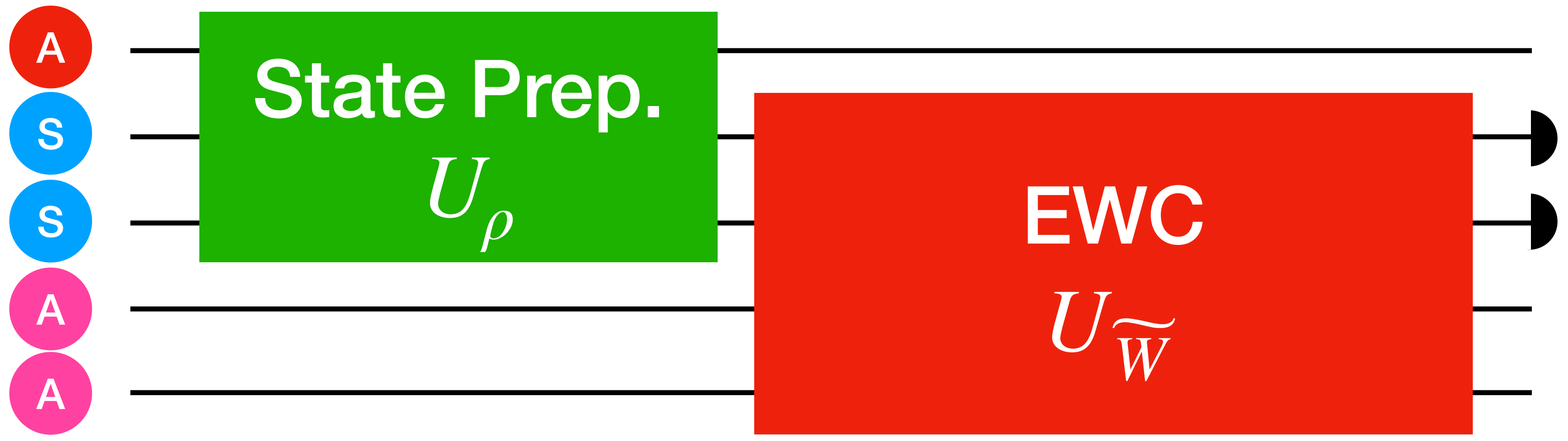


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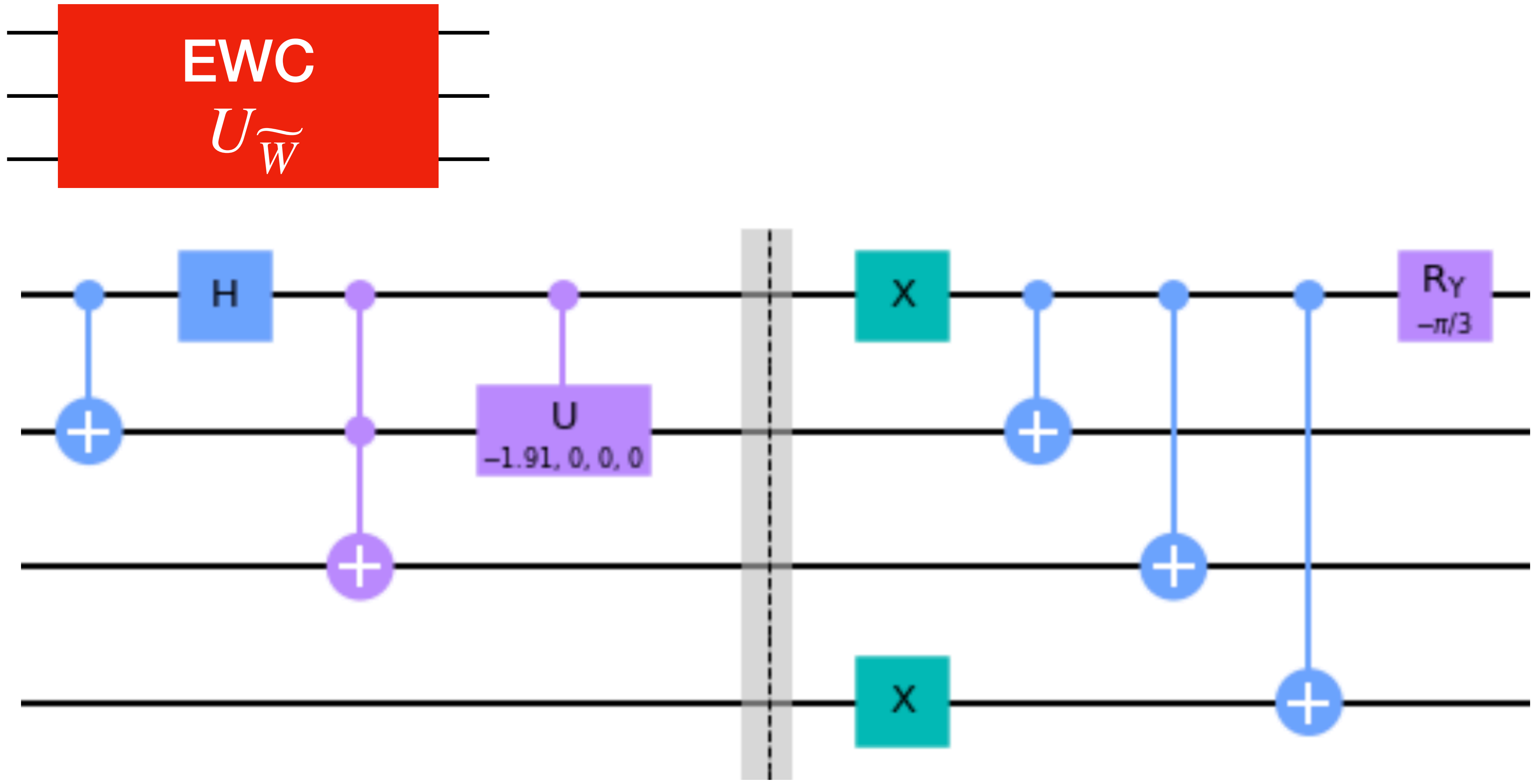


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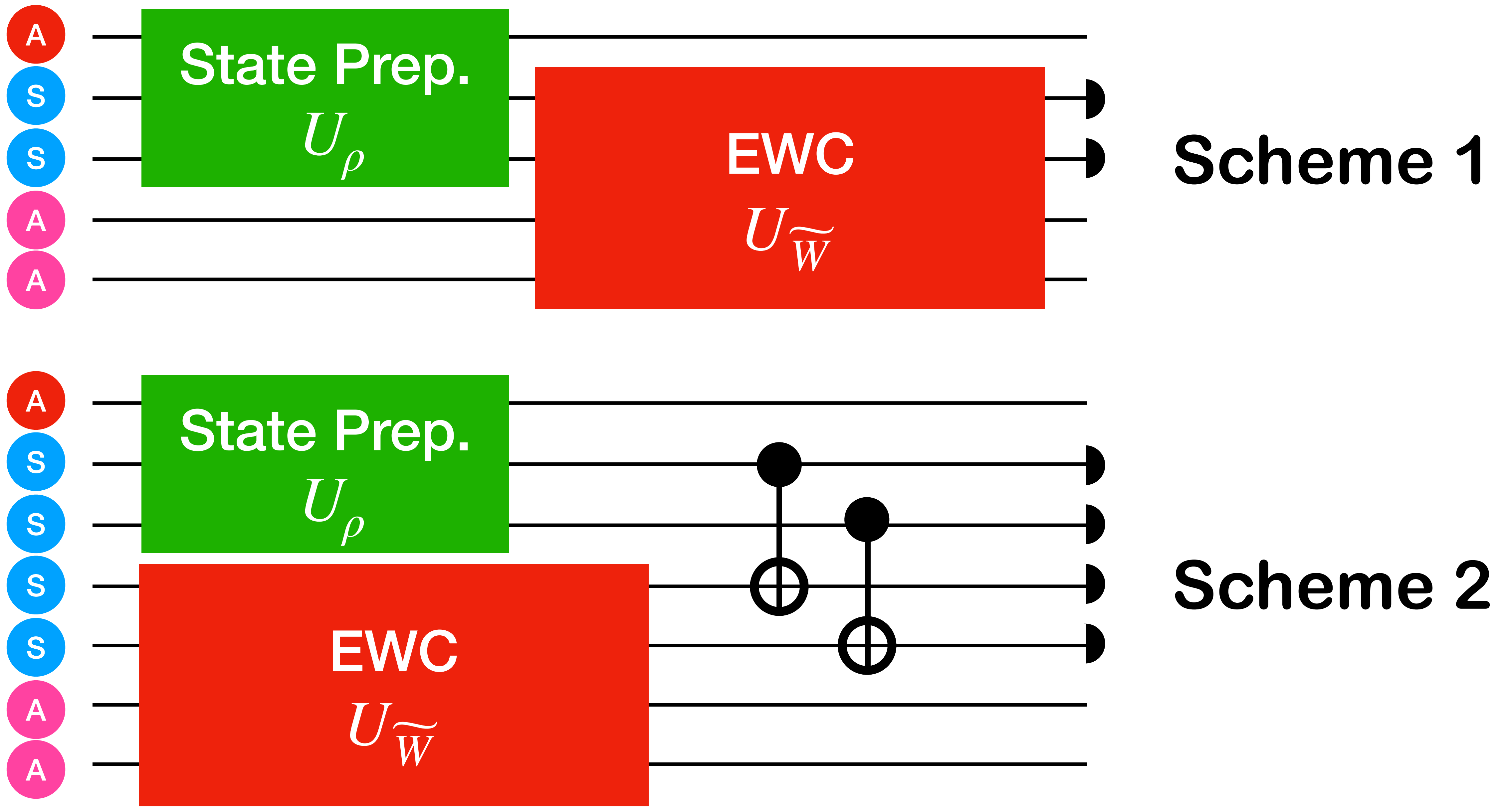
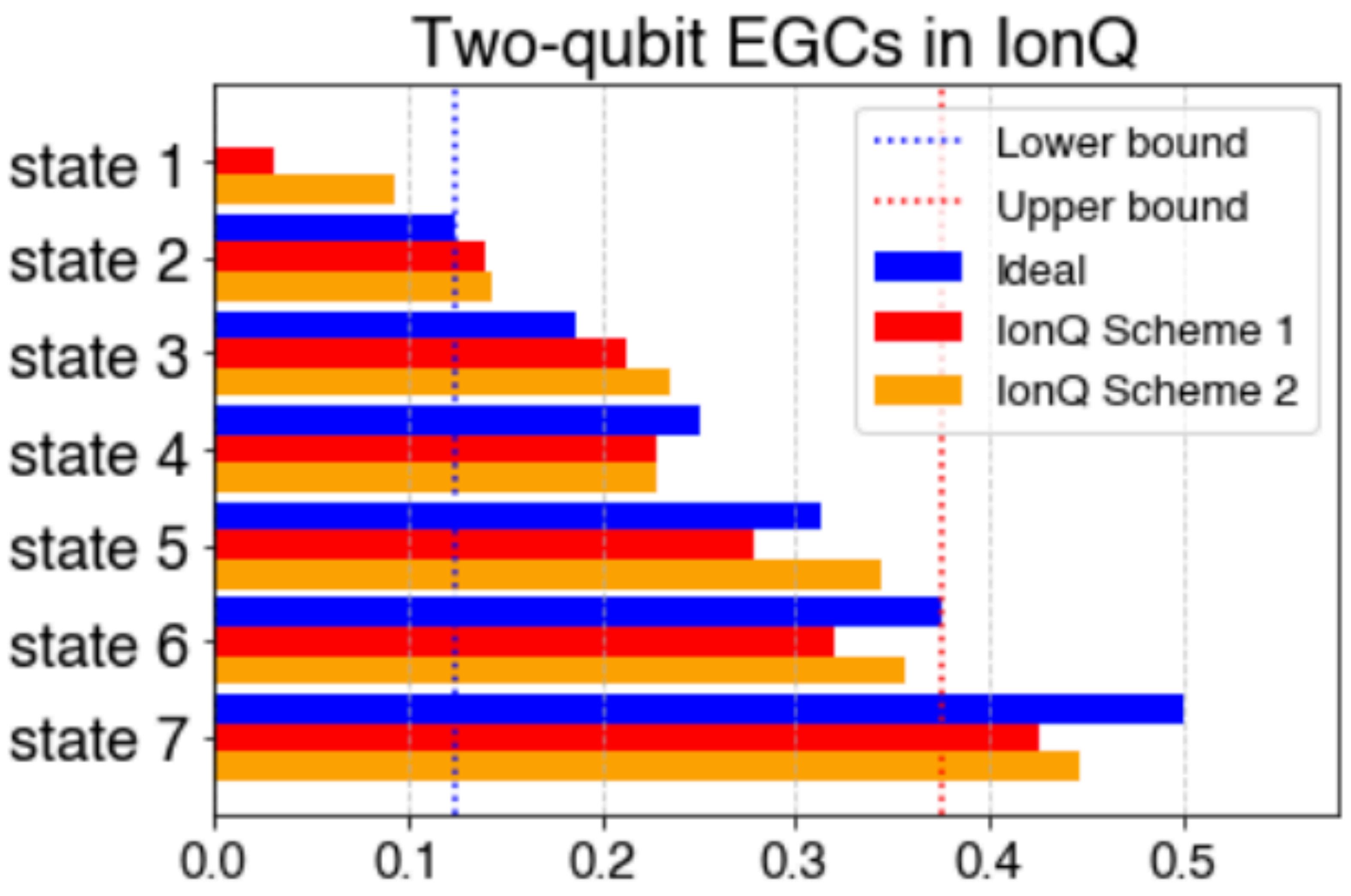
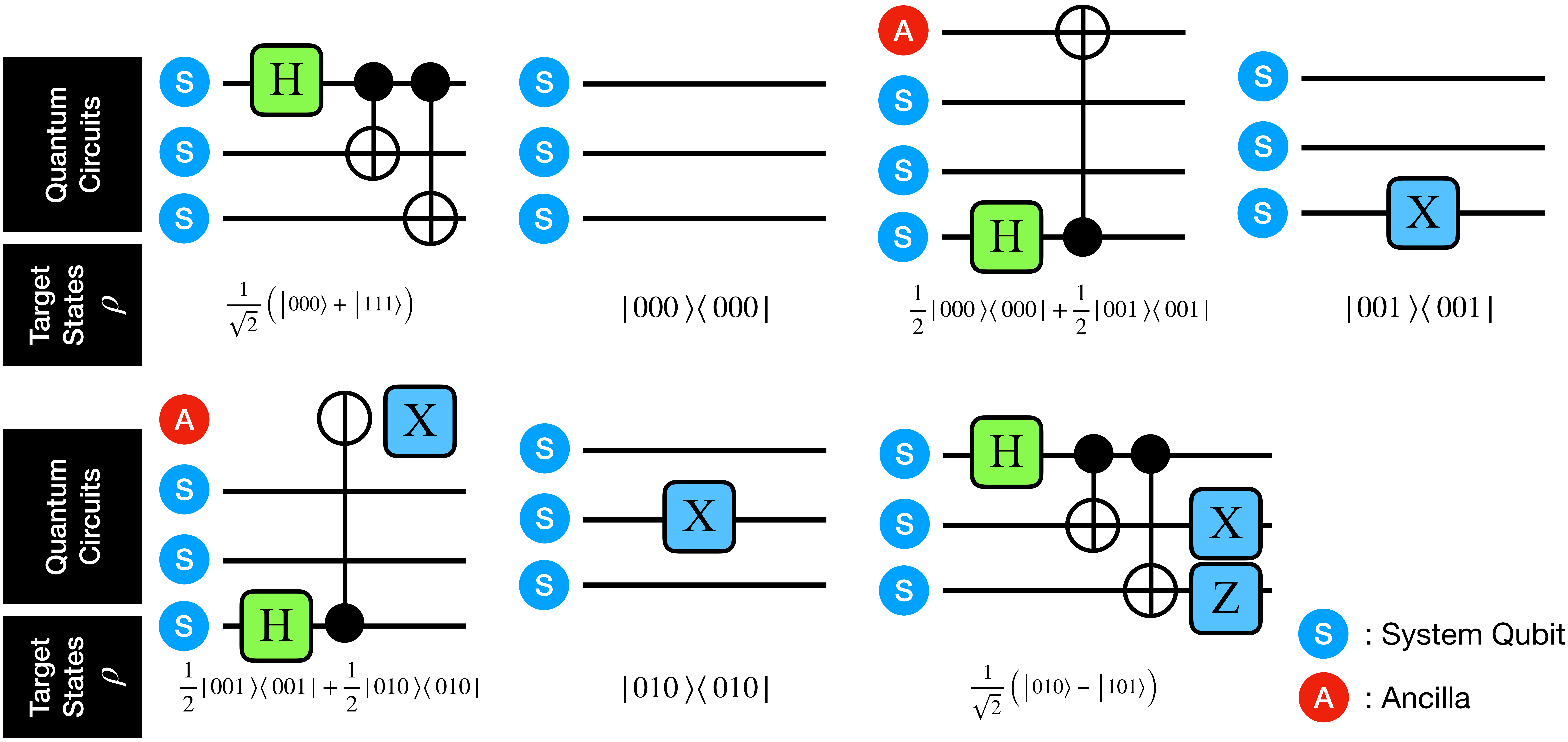
## 2-qubit demonstration in Ion Q

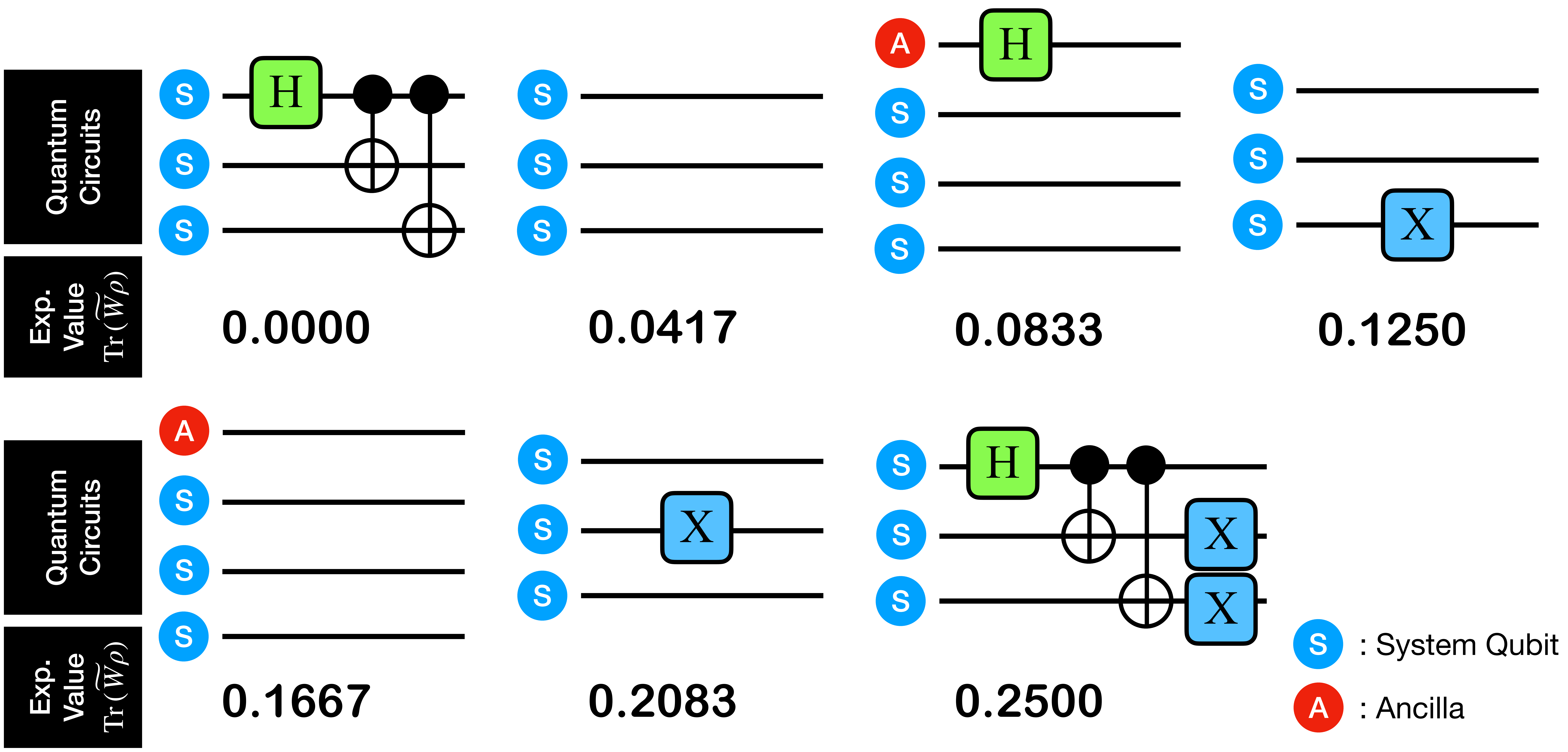


# Quantum Circuits with 3 qubits for demonstration



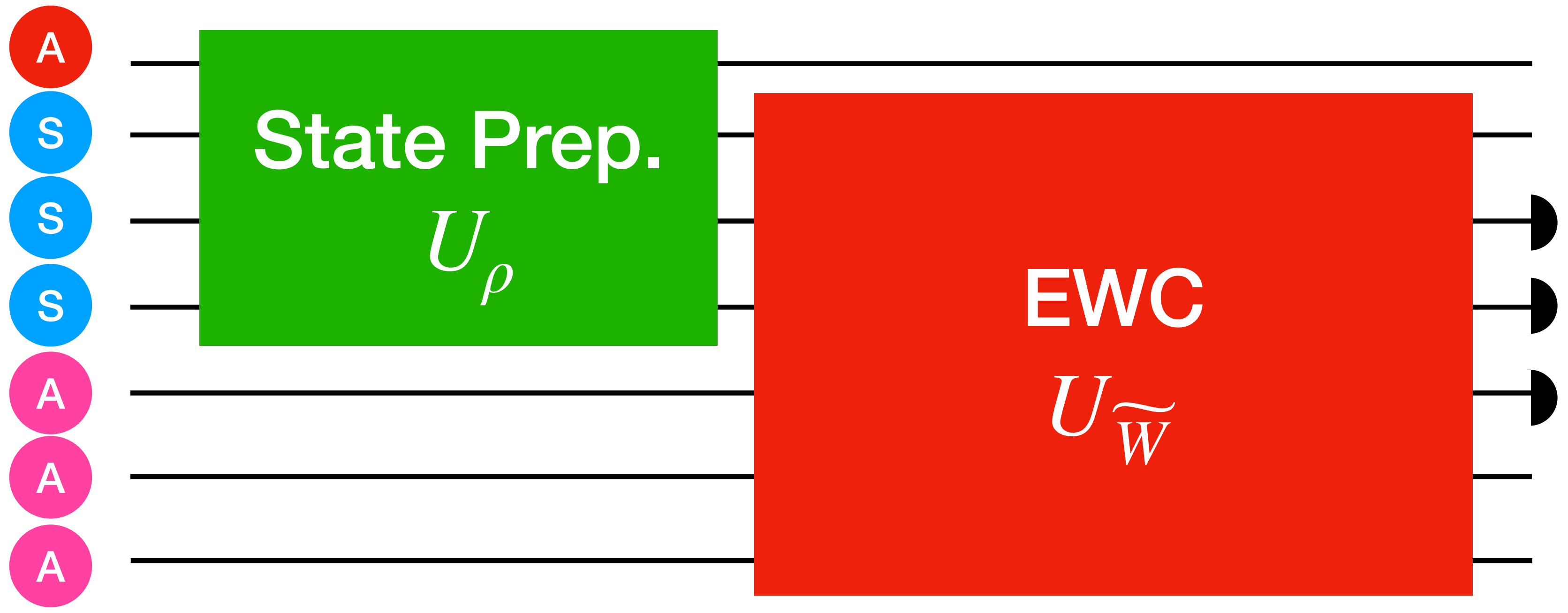
# Quantum Circuits with 3 qubits for demonstration



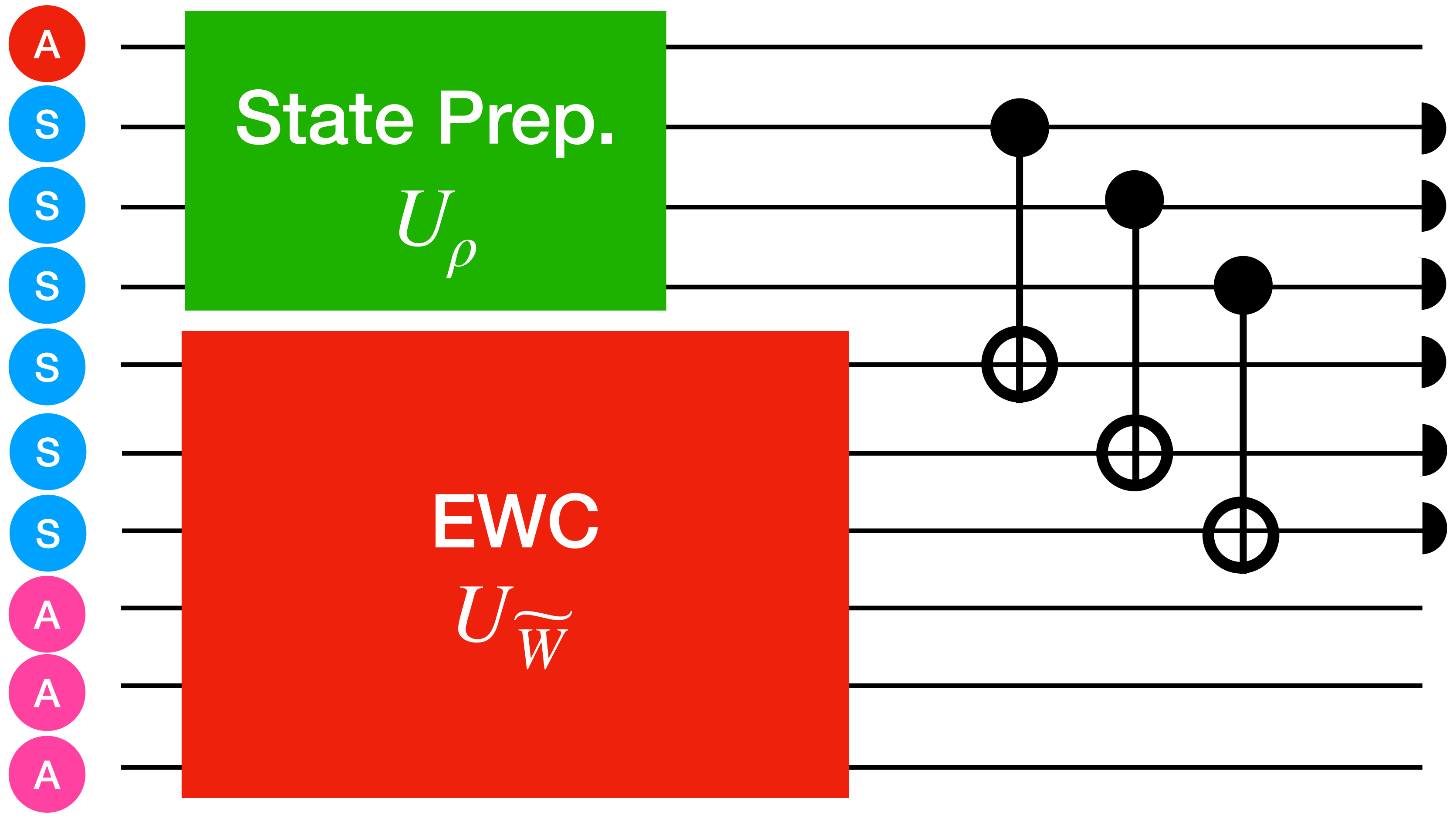

$$\widetilde{W} = \frac{1}{8}\left(I - (X_1X_2X_3 + Z_1Z_2 + Z_2Z_3)\right)$$


| | | |
|---|---|---|
| **State 1** | $\frac{1}{\sqrt{2}}\left(\lvert 000\rangle + \lvert 111\rangle\right)$ | **0.0000** |
| **State 2** | $\lvert 000\rangle\langle 000\rvert$ | **0.0417** |
| **State 3** | $\frac{1}{2}\lvert 000\rangle\langle 000\rvert + \frac{1}{2}\lvert 001\rangle\langle 001\rvert$ | **0.0833** |
| **State 4** | $\lvert 001\rangle\langle 001\rvert$ | **0.1250** |
| **State 5** | $\frac{1}{2}\lvert 001\rangle\langle 001\rvert + \frac{1}{2}\lvert 010\rangle\langle 010\rvert$ | **0.1667** |
| **State 6** | $\lvert 010\rangle\langle 010\rvert$ | **0.2083** |
| **State 7** | $\frac{1}{\sqrt{2}}\left(\lvert 010\rangle - \lvert 101\rangle\right)$ | **0.2500** |

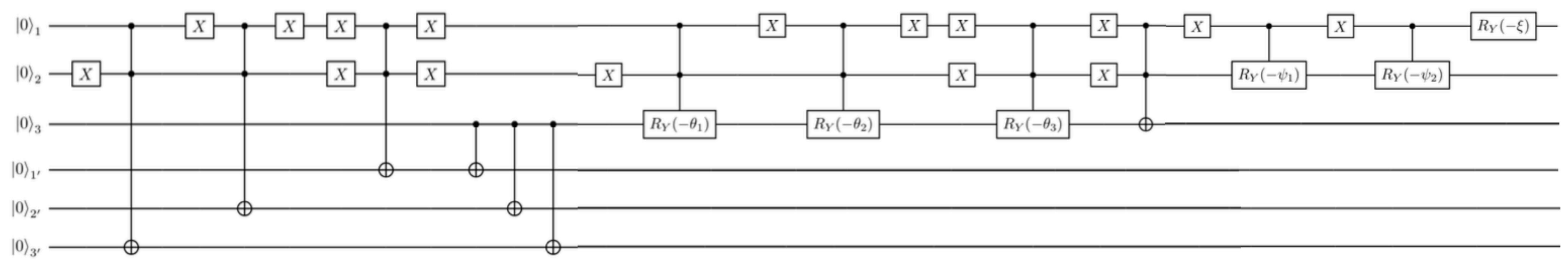
# 3-qubit demonstration in Ion Q




**Scheme 1**




**Scheme 2**


$$\widetilde{W} = \frac{1}{8}\left(I - (X_1X_2X_3 + Z_1Z_2 + Z_2Z_3)\right)$$


| | | |
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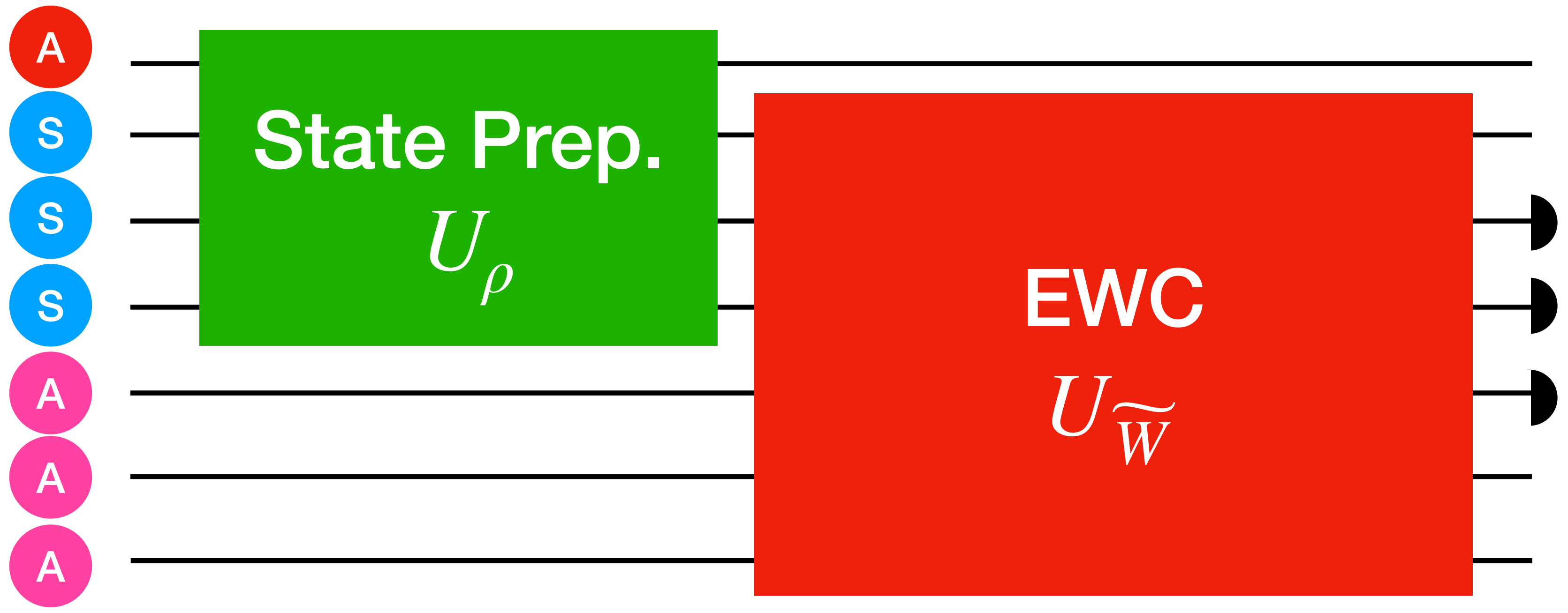
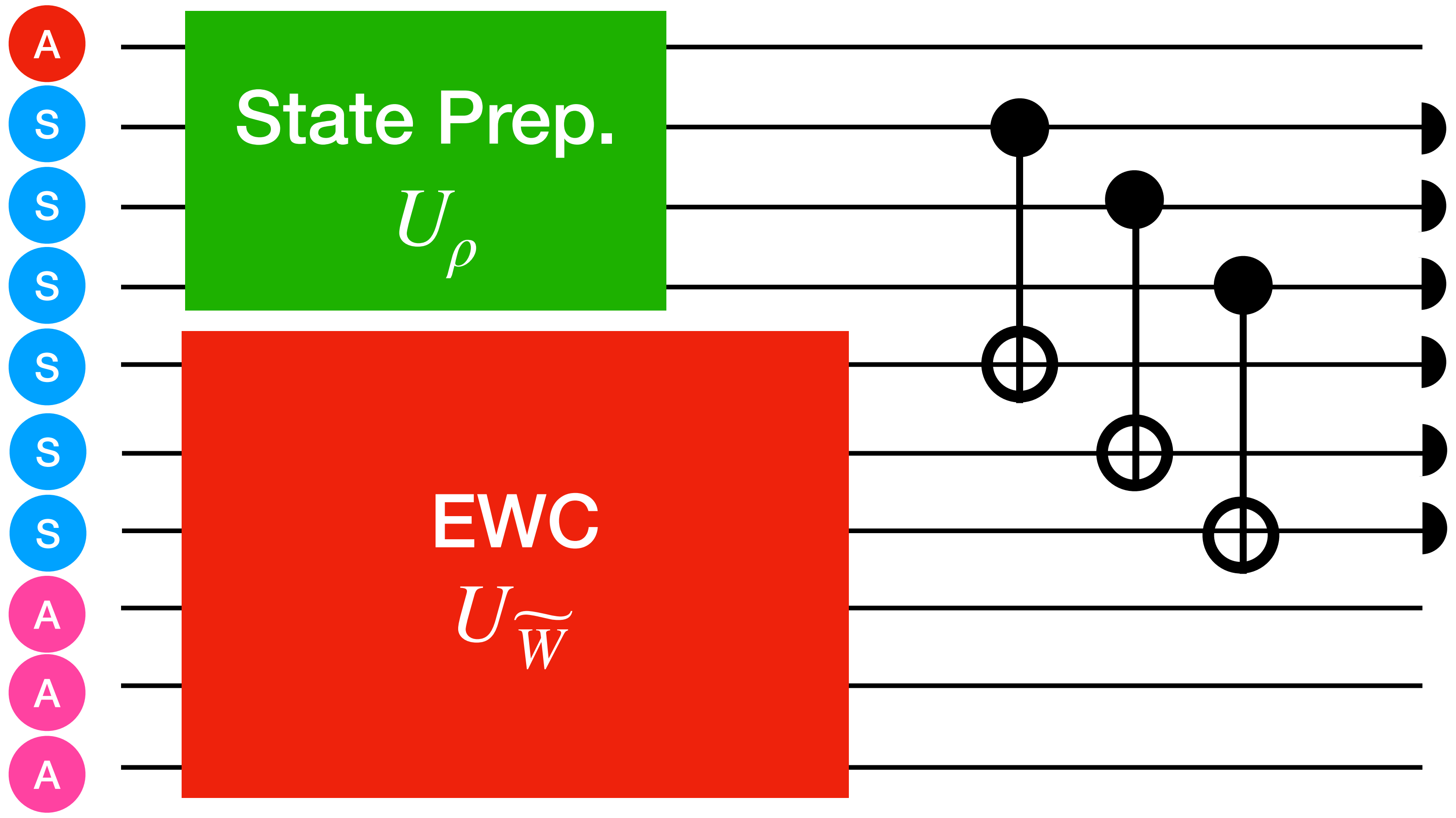
| **State 7** | $\frac{1}{\sqrt{2}}\left(\lvert 010\rangle - \lvert 101\rangle\right)$ | **0.2500** |

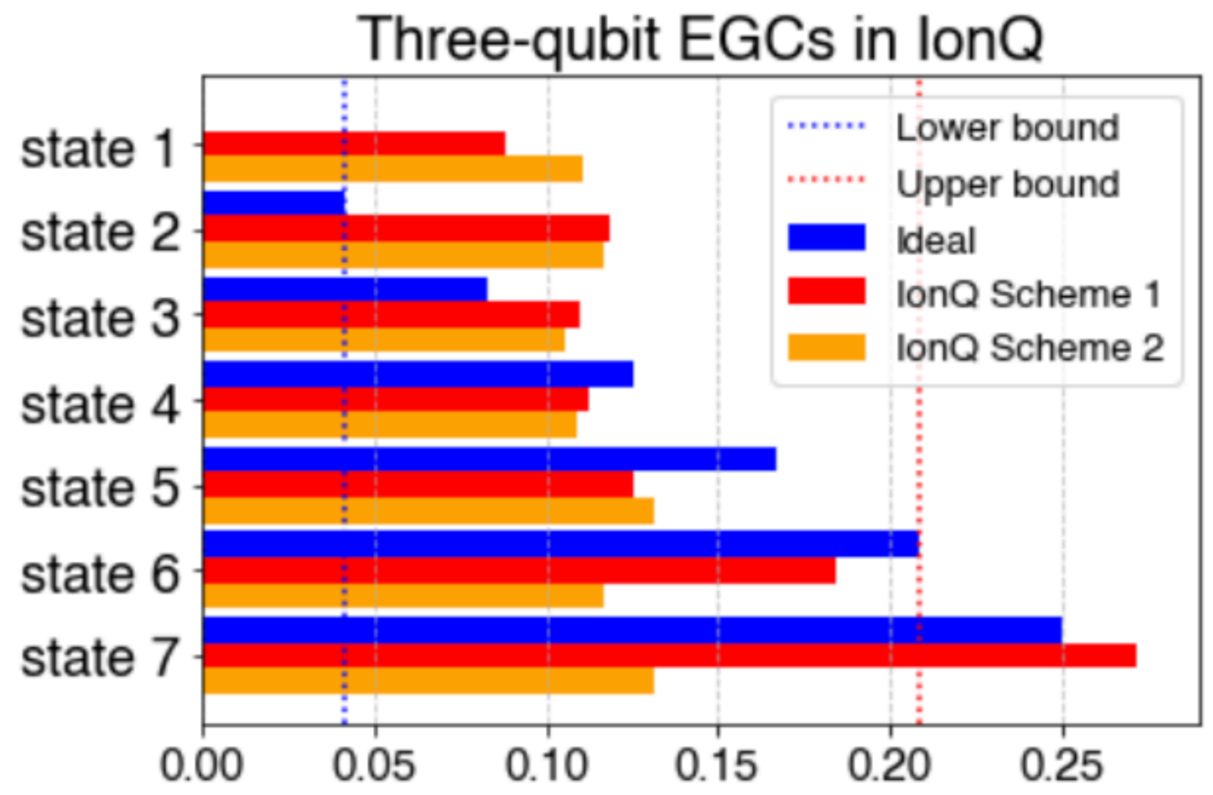
# 3-qubit demonstration in Ion Q




**Scheme 1**

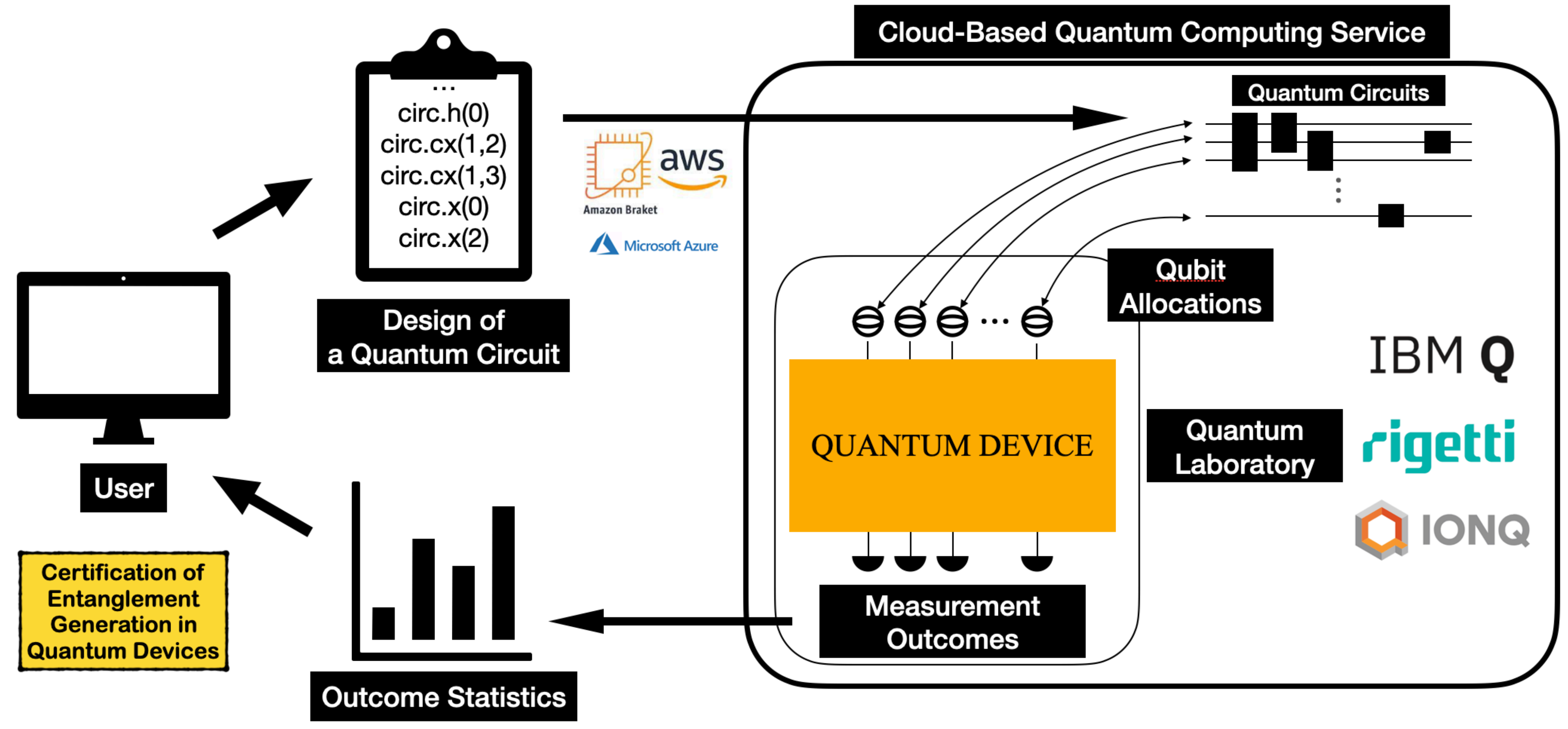


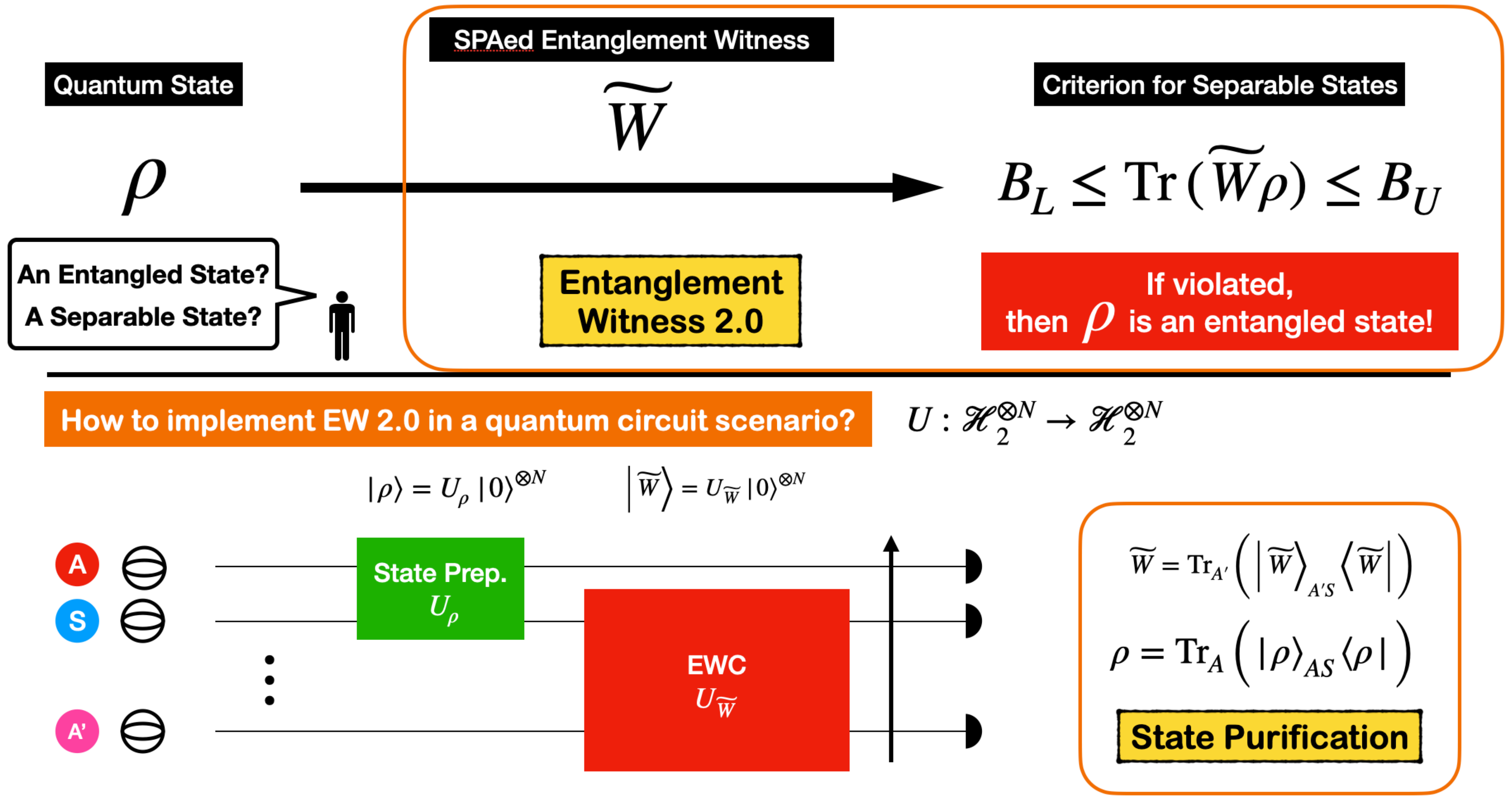

**Scheme 2**

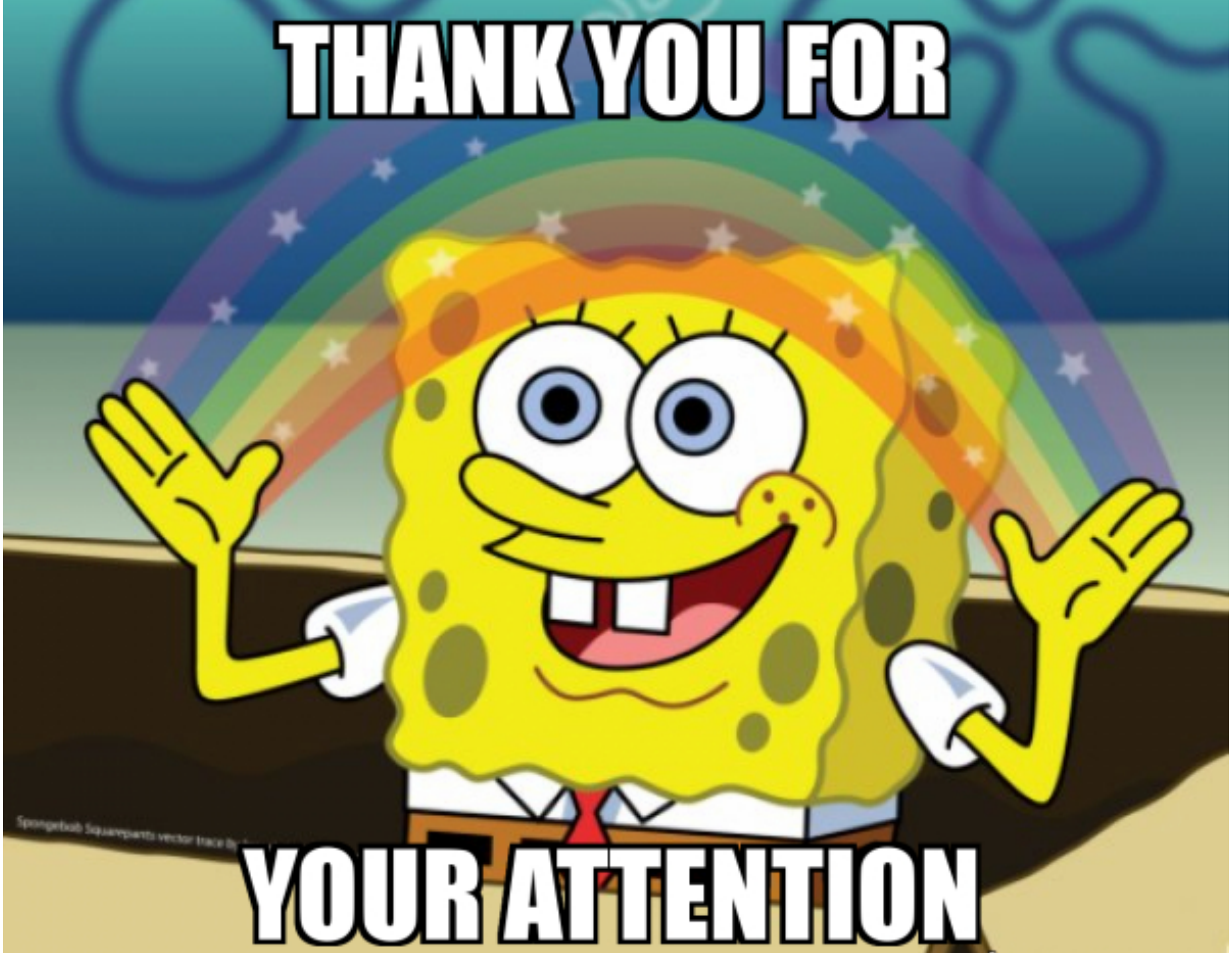


# Conclusions

- We have established the framework of certifying entanglement generation in a cloud-based quantum computing service **without the assumption of trusting qubit allocations.**
- The entanglement certification circuits are constructed by transferring **the framework of EW 2.0 to a quantum circuit model via SPA to EWs**.
- We have used the circuit architecture **to certify entanglement generation in IBM Q and IonQ services**.
- Our results **can be generally used to certify entanglement generation in a cloud quantum computing service.**




THANK YOU FOR
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